Comparison of the physical acoustic channel response of a line array of thin rectangular bars to an equivalent model of thin vibrating rectangular pistons.

Shane Chambers (1), Ralph James (2) and Alec Duncan (3)

(1) School of Physics, University of Western Australia
(2) School of Physics, University of Western Australia
(3) Centre for Marine Science and Technology, Curtin University, Western Australia

ABSTRACT

The resolution of an array is determined by the number and spatial distance of apertures (channels) within the array and the geometry of each aperture. The accurate design of acoustic sensing arrays relies on an *a prioiri* estimate of the expected far field radiation pattern of reciprocally behaved elements chosen for each aperture which is difficult to calculate under damped and loaded conditions. The estimated response of one channel of a vertical line array, when modeled as a series of rectangular vibrating pistons on a rigid baffle, is compared to the measured response of one channel of a line array comprised of a series of thin rectangular bars under load and operating off resonance. Although simple modeling can predict the 3dB main lobe width of the channel with some accuracy, loading and damping effects will alter the individual element response and hence the sensitivity of the array and side lobe magnitudes when off axis steering. This is important to note when estimating array gain and noise contributions from sidelobes under steered conditions.

INTRODUCTION

When designing a passive underwater acoustic array one has to consider the angular acoustic response of the elements within the array if the intended use is to digitally steer the array's receive aperture over a large angular range. The elements present a limiting factor on the angular sensitivity of the array as they envelope the array directivity function $H(\theta,\phi)$. Usually the elements are placed in a resin or some sort of rigid mould where the acoustic parameters of the surrounding damping material are not well known. Additional loading of the element in water when encased in housing and/or covered in a thin membrane layer presents more complexity when assessing element response. Given this, it is difficult to estimate what the angular responses of the elements are and one has to resort to either complex methods such as finite element modelling (FEM) or deal with simpler, conventional Huygens/Rayleigh based solutions. This paper looks at the measured singular channel response of a 32 channel vertical linear array of thin rectangular pistons and compares the results to conventional theory. Using the results of one channel the steering sensitivities are examined assuming all channels behave similarly.

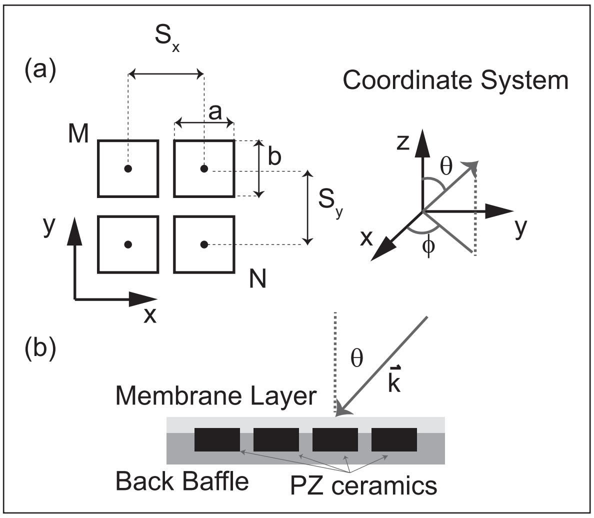

Figure 1a: Parameters used in the directivity functions. **1b**: Side view of ceramics encased in a baffle with a thin acoustically matched layer interfacing with the water.

$$H(\theta,\phi) = \frac{sin(NX)}{NsinX}\frac{sin(MY)}{MsinY} \tag{1}$$

where:

$$X = kS_X(sin\theta cos\phi - sin\theta_0 cos\phi_0)/2 \tag{2}$$

and:

$$Y = kS_Y(sin\theta sin\phi - sin\theta_0 sin\phi_0)\ /2 \tag{3}$$

Where M and N are the dimensions of the array, k is the wavenumber, θ_0 and ϕ_0 are the steering directions and S_X and S_Y are the X and Y distances between centres of the elements or apertures within the array (parameters are shown in Figure 1a).

THEORY

This paper is not intended to be a primer on the theory of acoustic arrays. Such a field is vast and there are many sources of information that give adequate and thorough treatment of array theory, to name a few (Burdic 1984), (Sherman & Butler 2007), (Naidu 2001). The equations used in the analysis will only be stated. It is suggested the reader refer to the above references for derivation of equations.

Array directivity function for an M × N omnidirectional array

The angular directivity function of a reciprocally behaving M × N array with omnidirectional elements lying in the XY plane as shown in Figure 1a can be shown to be:

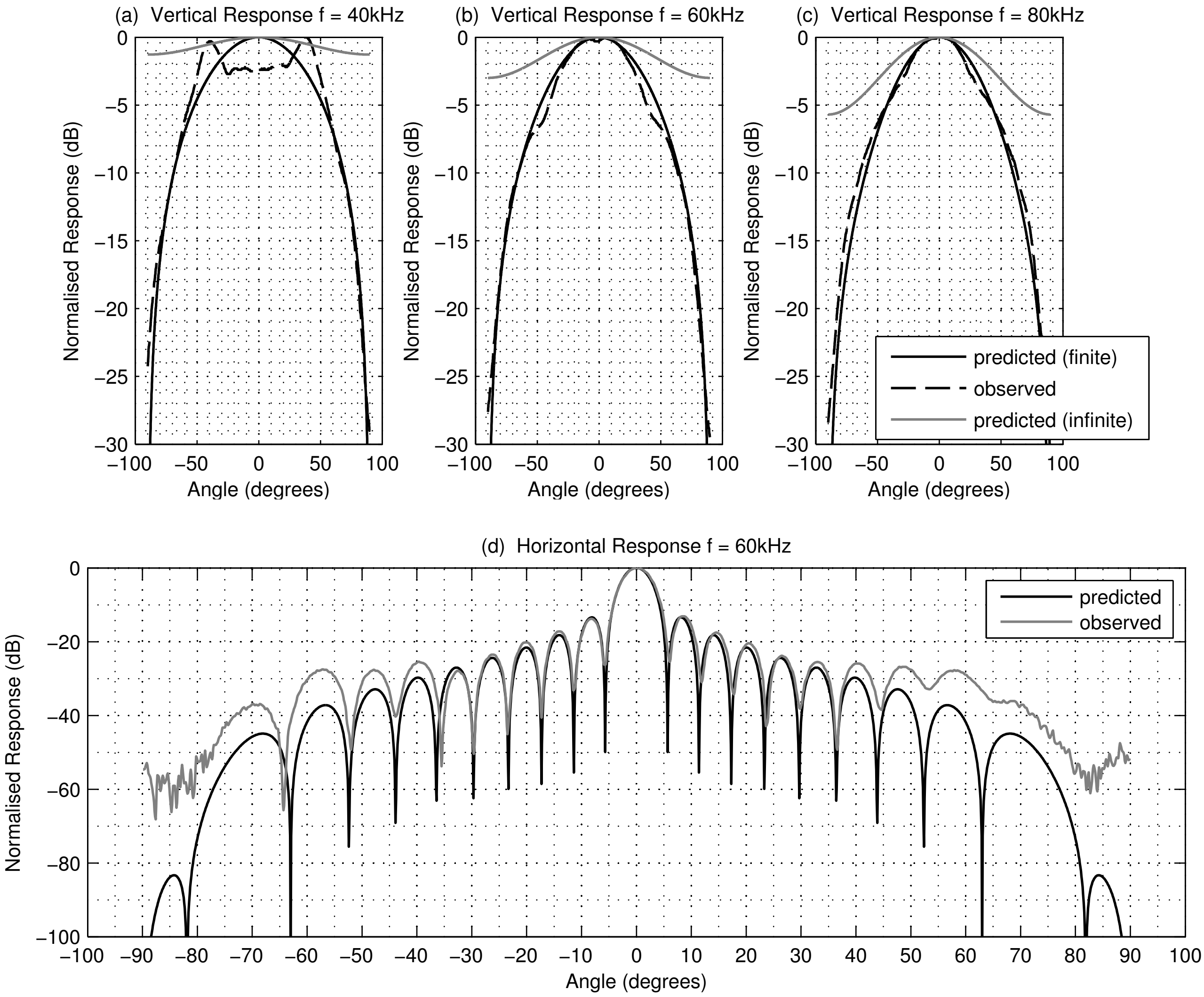

Figure 2a,b,c: Vertical channel element responses for 40kHz, 60kHz, 80kHz compared with the infinite and finite rigid baffle approximations. **2d:**Observed and estimated horizontal channel response of 20 thin rectangular pistons at 60kHz.

Far field directivity function of a thin vibrating rectangular piston in a finite rigid baffle

Calculation of the far field response of a thin vibrating rectangular piston usually considers the element surrounded by an infinite plane rigid baffle (Freedman 1960), (Ocheltree & Frizzel 1989), (Kinsler et al. 1999). In underwater acoustics the element is assumed to have one side separated from the water by a thin acoustically matched membrane and its baffle is considered finite. This assumption leads to the directivity function of a thin rectangular piston using the Rayleigh-Sommerfield diffraction formula (Selfridge, Kino & Khuri-Yakub 1980):

$$H_E(\theta,\phi) = \frac{sin\left[\frac{ka}{2}sin\theta cos\phi\right]}{\frac{ka}{2}sin\theta cos\phi}\frac{sin\left[\frac{kb}{2}sin\theta sin\phi\right]}{\frac{kb}{2}sin\theta sin\phi}cos\theta \qquad (4)$$

Where a and b are the X and Y dimensions of the rectangular piston elements (Figure1a).

The main difference between a finite and infinite baffle is the appearance of the $cos\theta$ term on the far right hand side of equation 4.

Combined directivity function for an M × N array of equally spaced thin rectangular pistons

Since in the far field all rays are parallel the product theorem is applicable when considering the combined directivity function of the array $H_C(\theta,\phi)$, and is the product of the omnidirectional array and element directivity functions:

$$H_C(\theta,\phi) = H(\theta,\phi)H_E(\theta,\phi) \qquad (5)$$

METHOD

A preliminary channel calibration of the array took place on February 4, 2012 at the facilities of Lake Kelk, Neptune Sonar Plc, Yorkshire, UK. The results of one central channel, channel 16, are presented.

The receiver consists of a 32 × 20 matched rectangular element array where each horizontal cluster of 20 elements is grouped into one channel. Element spacing within each channel has been optimised for good sensitivity and steering capability at a design frequency bandwidth 60kHz to 80kHz and for off resonant performance to reduce the effect of inter element coupling. The array response was measured with a pre-calibrated composite transducer and using the free field calibration by comparison method (IEC 2006).

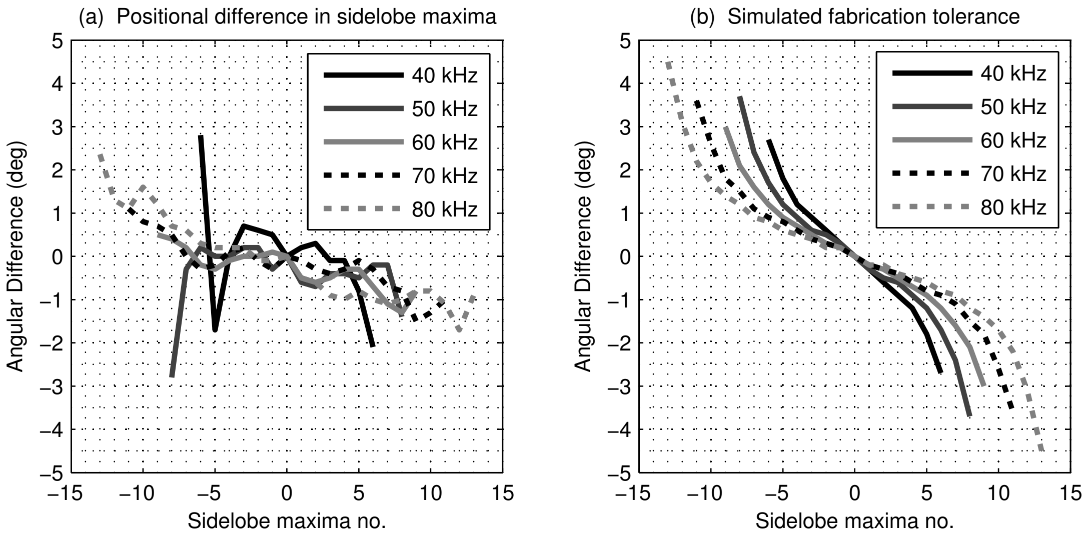

Figure 3a: The difference in positions of sidelobe maxima calculated by: estimate position - observed position. **3b:** The difference in positions of sidelobe maxima when modelling a fabrication tolerance of ± 0.15mm.

The positioning of the array and hydrophone was at a sufficient far field distance with no reflection interference. The measurements took place in fresh water at a temperature of 4° Celsius, depth 2.5m and at frequencies 40kHz, 50kHz, 60kHz, 70kHz and 80kHz.

The angular patterns were produced by rotating the array in 0.3 degree increments. Since the source strength was known, results were reduced to dB rel. 1V/µPa @1m. The uncertainty in measurements is less than ± 1dB.

RESULTS

Channel Directivity Response

The observed and estimated vertical channel directivity functions at 40kHz, 60kHz and 80 kHz are shown in Figures 2a, 2b and 2c and the horizontal channel directivity function at 60kHz is shown in Figure 2d. The magnitudes are relative to the level of the signal received at 0° (broadside) or at the angle of maximum response.

Since the width of a vertical channel is the width of one element, the vertical response is equivalent to the vertical directivity function of a single element. The predicted plots with and without the Rayleigh Sommerfield correction for finite baffles are shown. The inclusion of the $cos\theta$ term in the calculation results in a fairly good agreement with the observed results and is in line with a past observation elsewhere (Selfridge, Kino & Khuri-Yakub 1980). The extra term has then been used for all other estimations of array directivity functions.

The horizontal directivity function of one channel is the directivity function of 20 equally spaced elements in an array configuration and is shown in Figure 2d. There is increasing divergence between estimated and observed positions of sidelobe maxima as the sidelobe order gets larger. To investigate this divergence, the positions were numerically calculated for all calibration frequencies. The results are plotted in Figure 3a. The results illustrate the magnitude of the divergence in the positions as the order of sidelobe maxima increases. The angular positions of the observed sidelobe maxima were less than the estimated position of sidelobe maxima and this difference appears to behave linearly for low sidelobe maxima. This linear behaviour can be explained by modelling the 20 element array with the specified fabrication tolerance of 0.15mm. The range in divergence of each sidelobe maxima over the horizontal centre spacing range S_X ± 0.15mm was numerically calculated. The result of this calculation is shown in Figure 3b. The trend is very similar to what has been observed in Figure 3a, and within a similar angular range for sidelobe maxima order -5 to 5. This similarity indicates that the linear divergence of the positions of the observed and estimated sidelobe maxima is likely due to precision tolerances of positioning the elements during the fabrication process.

The difference in the estimated and observed horizontal sidelobe maxima magnitudes is shown in Figure 4. There is a significant difference between the estimated and observed sidelobe maxima as the order of the maxima increases. A negative value means that the magnitudes observed are higher than the magnitudes predicted. Figure 4 indicates that most of the observed fringe maxima magnitudes were higher than predicted for all measured frequencies, some quite significantly, up to 15dB. Since the magnitude of the sidelobe maxima is determined by the element directivity function $H_E(\theta,\phi)$ one would need to measure the horizontal angular response of a single element within the channel in order to differentiate the roles of fabrication tolerance and element directivity on channel response. Unfortunately a single element response was not measured, however the large differences ranging from 5dB to10dB for the outer fringes are noteworthy. When estimating the signal to noise characteristics of the array the magnitudes of the sidelobes are an important factor to consider. The higher observed magnitudes of the sidelobe maxima imply there is a physical reduction in the noise suppression performance from the initial array design.

The difference in estimated and observed beamwidths both at the 3dB at 10dB points is illustrated in Figure 5. It is evident that the modelling of the horizontal beamwidths does agree quite well (within 5%), with the observations of beamwidths at all frequencies. The absence of positive percentage differences means that the estimated 3dB and 10dB beamwidths were slightly narrower than the observed beamwidths for all frequencies. When considering the vertical beamwidths there is a large variation in differences. This can be attributed to the estimations of the vertical element response and the differences in beamwidths observed in Figures 2a, 2b and 2c.

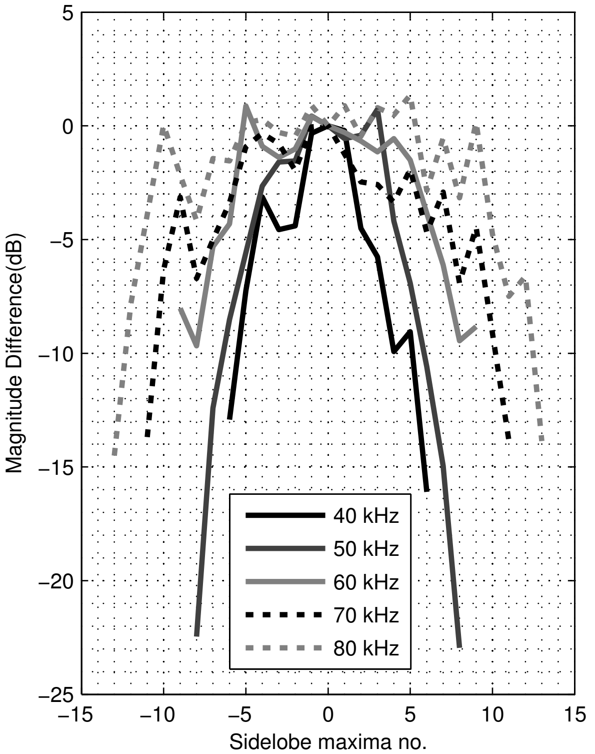

Figure 4: The difference between estimated and observed horizontal sidelobe maxima for all calibration frequencies.

Beam Steering Response

The effect of the element on the vertical beam steering response is shown in Figure 6. The difference between estimated and observed mainlobe magnitudes are plotted against steering angles from broadside to 45°. This is the operational range of the array and grating lobes for higher frequencies start to appear past this range limit. The results were obtained by simulating the steering of the beam using the observed vertical directivity function of one channel and assuming all channels in the array behaved similarly. The simulation was compared to steering the mainlobe using the estimated vertical directivity function for a single channel. A positive number implies that the estimated magnitude of the mainlobe is greater than the observed mainlobe.

At 40kHz the significant difference both at broadside to 15° and 35° to 45° range can be attributed to Figure 2a where there is a marked difference in the observed vertical directivity of the channel, and is actually greater in the range 35° to 45° than at broadside. For other frequencies the divergence between estimated and observed mainlobe magnitudes in the 25° to 55° range in Figures 2b and 2c affects the steering sensitivities up to 2dB.

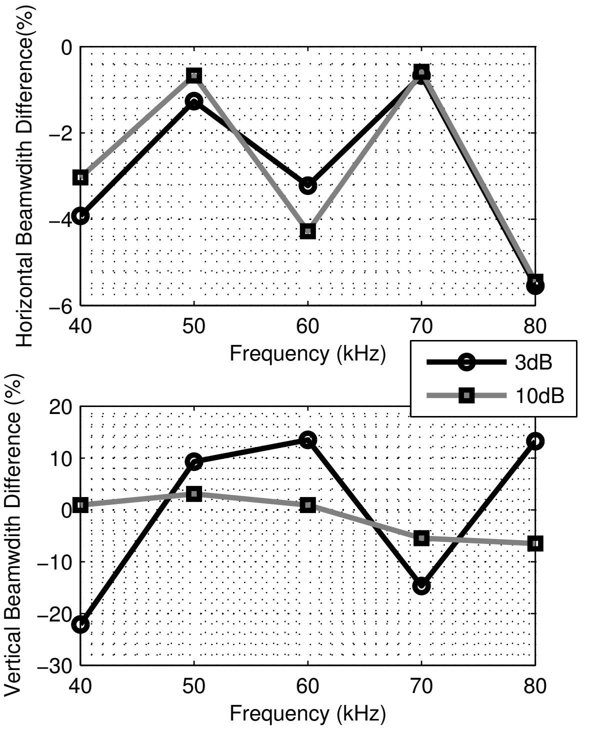

Figure 5: Vertical 3dB and 10dB beamwidth percentage errors for one channel. Calculated by: estimated beamwidth - observed vertical beamwidth

DISCUSSION

Array modelling using the Huygens/Rayleigh solutions is surprisingly accurate when considering that the positions of sidelobes has been shown to be largely dependent on tolerances in fabrication. The weakness in the design of an array when considering steering performance is estimating the element response.

The shaping of the side lobe maxima is an important consideration when designing a beamforming array with specified signal to noise characteristics. Adequate noise suppression must be traded off with the loss of sensitivity when steering. The spurious divergence away from the modeled vertical responses in the 25° to 45° range is hard to predict and it is unlikely that complex FEM techniques would be able to predict such a divergence. If a flat and/or predictable sensitivity response is required across a specific steering range it would be advantageous to measure the directivity functions of a singular element before embarking on a costly fabrication process.

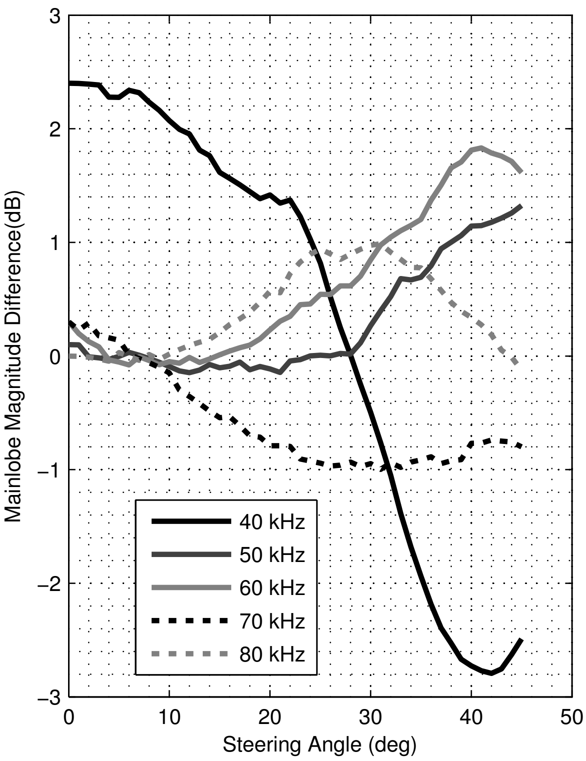

Figure 6: The effect of the element response on the steered mainlobe of the array in the vertical YZ plane.

CONCLUSION

The objective of this paper was to compare the observed and estimated vertical and horizontal channel response of an underwater acoustic array. It was found that if the elements are considered to be vibrating thin rectangular plates behaving like pistons in a finite rigid baffle that this assumption is sufficient to predict the beamwidth positions and the magnitudes of the low order sidelobes to a good degree of accuracy using Huygens Rayleigh based modelling. There is a divergence between estimated and observed magnitudes and positions of sidelobes and this difference becomes significant at large angles. The divergence in positions of higher order sidelobes are affected by fabrication tolerances, whilst the magnitudes are affected by inaccurate estimates of the element response. This inaccuracy will affect the expected sensitivity when steering the array to the peripheral ranges, in this case up to 2 dB for the range of design frequencies for this particular array.

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