STATISTICS AND THE TWO MICROPHONE METHOD FOR THE MEASUREMENT OF SOUND ABSORPTION COEFFICIENT

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Abstract

When calculating the normal incidence absorption coefficient using the two microphone method (ASTM E 1050 - 98) a number of different approaches can be taken to estimate the statistical variation of different samples of the same material. The most obvious approach can sometimes produce results that violate physical principles, especially for highly absorbent or highly reflective materials. This paper describes a method to estimate the statistical reliability of the procedure, which is both simple and based on sound physical principles, giving the range of absorption coefficients that would be expected to be measured 95% of the time. It is suggested that this method is adopted for future two microphone measurements when the statistical uncertainty of the measurements is to be reported.

Nomenclature

α	normal absorption coefficient
R	complex amplitude reflection coefficient
H	calibrated transfer function
k	wavenumber
d	microphone spacing
μ	true mean
$\bar{x}$	mean of the set measured
x_j	value of the j^{th} measurement
n	number of measurements
σ	true variance
s	standard deviation
t_n	Students t-distribution
H_j	j^{th} sampled transfer function
y_j	sample constant in the range [-1,1]

Introduction

The absorption of sound by acoustic linings is an important aspect of engineering noise control [1]. There is no general method available to calculate this absorption coefficient from the properties of the lining, although it is possible to predict the absorption coefficient of a specific class of materials [2]. Experimental measurement of the absorption, and other related properties, is required in most cases and there are a number of international standards available.

One widely used standard, AS/NZS 1935.1:1998 [3], prescribes the determination of the absorption coefficient by the measurement of the maximum and minimum sound pressure levels of a standing wave in a tube with a loudspeaker at one end and a sample of the material under test at the other end. The measurement is accurate, but can be time consuming because each measurement can only be performed at a single frequency. An alternative technique, ASTM E 1050-98 [4], the "Standard test method for impedance and absorption of acoustical materials using a tube, two microphones and a digital frequency analysis system" is preferable because measurements can be made over a range of frequencies simultaneously. The two techniques have been shown to produce similar results [5] and the two microphone technique has become widely used.

While there have been a number of studies investigating the influence of errors on the measurements [6,7], no studies have been found that show how to estimate the statistical variation of absorption coefficient for different samples of the same material. A direct application of uncertainty estimation to the calculated normal absorption coefficient can sometimes produce results for the 95% confidence interval that are over one or below zero. This would mean that either more energy returns from or is absorbed by the sample than was incident upon it, thus violating physical principles. The ISO guide to certainty in measurement [8] offers a way to estimate the uncertainty, but it appears to be difficult to apply to the two microphone technique. Hence there is a need for a simple method, based on sound physical principles, that gives the range of absorption coefficients that would be expected to be measured 95% of the time.

Theory

In a similar way to the standing wave ratio method of sound absorption measurement, the two microphone technique uses a tube with a sound source at one end and the sample placed at the other (Figure 1). A broadband signal is applied to the sound source and the transfer function between two microphones, specially calibrated to minimise amplitude and phase errors between the channels, is measured. Provided only plane waves propagate in the tube past the microphones, the analytical transfer function between the two stations allows the decomposition of the field into forward and backward travelling waves and hence the determination of acoustic absorption and other related properties.

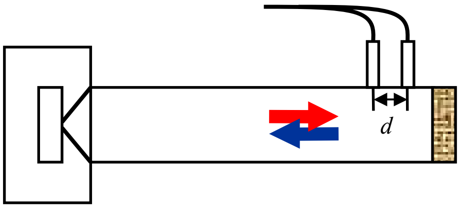

Figure 1. Schematic of a two microphone test with the sound source at one end, the sample at the other and the two microphones spaced at known distance d.

The normal absorption coefficient, which is the fraction of the total incident sound power that is absorbed by the material at the end of the tube, is defined [4] as:

$$\alpha = 1 - |R|^2$$

$$|R| = \left| \frac{H - e^{-jkd}}{e^{jkd} - H} \right| \tag{1}$$

where R is the complex amplitude reflection coefficient, which is defined as the complex ratio of incident to reflected wave amplitudes, H is the calibrated transfer function between microphones 1 and 2, $j = \sqrt{-1}$, k is the wavenumber and d is the microphone spacing. For passive materials, the absorption coefficient is physically limited to a value that lies between 0 and 1.

The estimation of uncertainty in the measurement of a parameter can be made from a small number of measurements, if the assumption is made that the underlying distribution is normal. The true mean, μ,of a measurement can be estimated from the mean, $\bar{x}$, of the set measured where:

$$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n} \tag{2}$$

x_j is the value of the j^{th} measurement, and n measurements are taken. An estimate of the true variance, σ, of the distribution can be made by the standard deviation, s, of the set measured where:

$$s = \sqrt{\frac{\sum_{j=1}^{n} (x_j - \bar{x})^2}{n-1}} \tag{3}$$

If many measurements were made, and were found to be normally distributed, then $\mu = \bar{x}$, $\sigma = s$ and an interval, $\mu \pm 1.96\sigma$, could be formed that would contain 95% of all possible values of the measurement.

As we are interested in the variation in acoustic absorption of a particular material, a number of representative samples of the material must be measured using a complete two microphone test. In practice, the number of samples for which measurements of acoustic absorption can be made within a reasonable time is small ($n \approx 5$) and the interval that would contain 95% of all possible values of the measurement can be estimated by:

$$\bar{x} \pm t_n s \tag{4}$$

where t_n is taken from the Students t-distribution. Values of t_n for a given number of measurements can be found in books on statistics [9] or can be calculated using the tinv Matlab function from the statistics toolbox via:

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tn=-tinv((1-0.95)/2,n-1);                    (5)
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Simplistic approach

Measurements of one-third octave band normal absorption coefficients have been obtained for a particular material. One absorption measurement was made using the two microphone technique for each of the 5 representative samples of the material. The mean and standard deviation of the set were calculated using Equations (2) and (3) and the 95% confidence interval estimated using Equations (4) and (5).

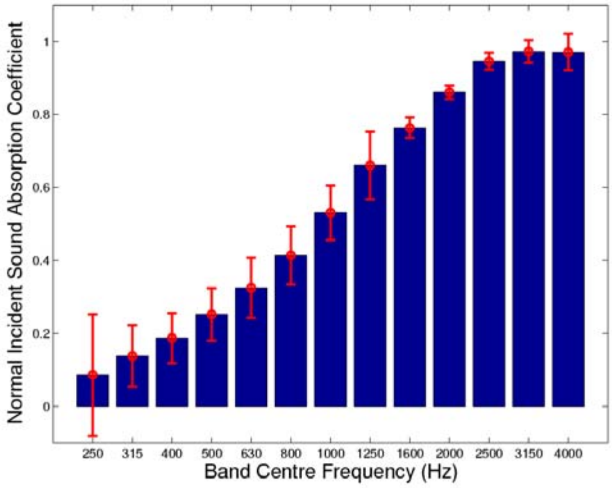

Figure 2. One-third octave band absorption coefficients mean and 95% confidence intervals estimated using the simplistic approach.

Figure 2 displays one-third octave absorption coefficients with the 95% confidence intervals superimposed. As can be seen, some of the calculations of the limits lie outside the range of physical validity of absorption coefficients (i.e. above 1 or below zero), and this approach can be considered to be somewhat naive.

Alternative method

The problem with the simplistic approach is that the transfer function between the two microphones (H) is being measured but the statistics of a parameter derived from this measurement (α) is being estimated by the transformation defined in Equation (1). The assumption of a normally distributed measurement of *H* is probably appropriate, but the assumption of a normally distributed absorption coefficient is probably not.

A brief literature review revealed little practical help. "The ISO guide to the Expression of Uncertainty in Measurement", as outlined in Bentley [8], offers a way to calculate uncertainty in measurement, but it appears to be difficult to apply to this case.

A solution has hence been formulated that satisfies the physical constraints of energy balance. As in the simplistic approach, a single two microphone measurement on each of the 5 representative samples was made. Instead of calculating mean and 95% confidence limits for normal absorption coefficients, these statistics are calculated for the measured transfer function *H*. These statistics represent a normal distribution of *H* about a mean spectrum. A representative spectrum was calculated from this distribution using

$$H_j = \bar{x} \pm t_n y_j s \tag{6}$$

where H_j is the j^{th} calculated transfer function and y_j is in the range [-1,1]. This calculated spectrum will be expected to lie within 95% of all measurements of *H*.

Figure 3 shows narrowband results for a range of calculated spectra that cover 95% of the measurements by varying y_j linearly between -1 and 1 in 41 steps.

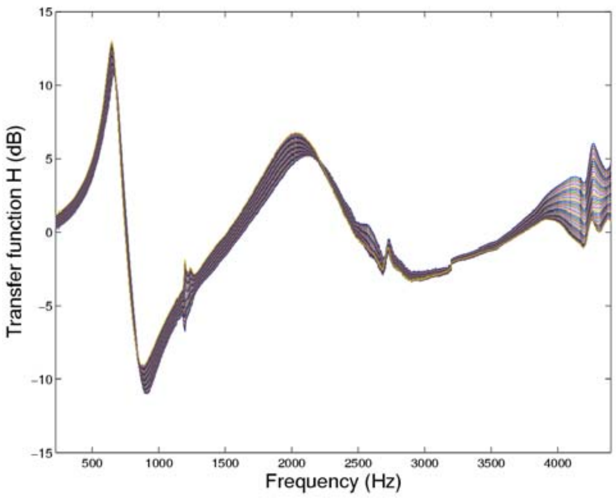

Figure 3. Calculated spectra *H* that lie between the 95% confidence intervals.

The range of transfer functions was then used to calculate a range of values of normal absorption coefficient. If the range of transfer functions is a good representation of all values between the 95% confidence intervals for *H*, then the range of calculated normal absorption coefficients will also cover 95% of all possible values of the measurement of α.

Figure 4 shows values of normal absorption coefficient calculated from the range of transfer functions shown in Figure 3. The resulting envelope of values will cover 95% of the values of normal absorption coefficient. Figure 5 shows a zoomed plot of the normal absorption coefficient values. In both Figures 4 and 5 no value of absorption coefficient is above 1, as dictated by physical considerations.

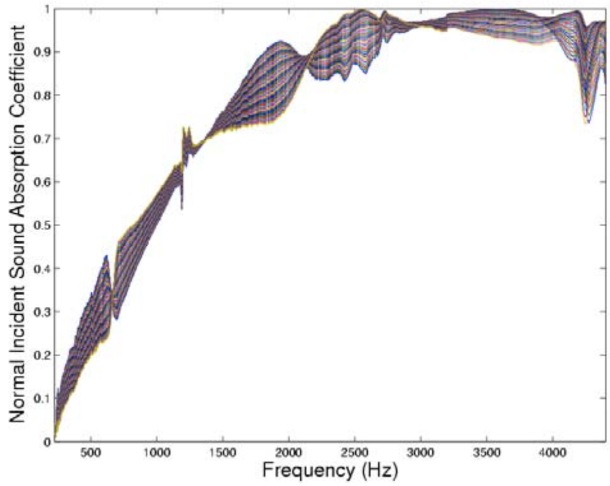

Figure 4. Normal incidence sound absorption coefficient calculated for a series of transfer functions uniformly distributed between the 95% confidence limits.

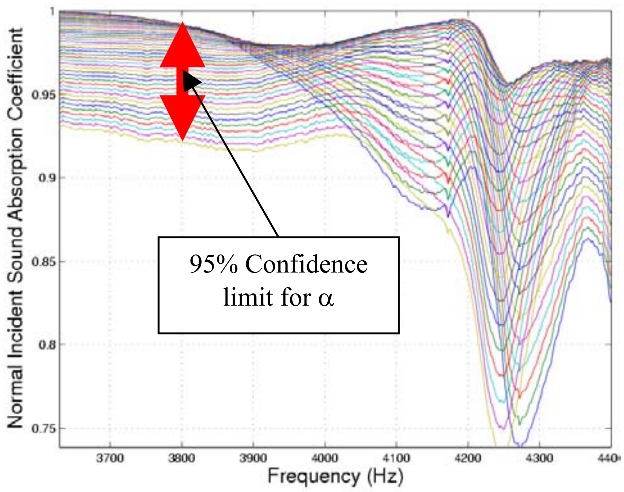

Figure 5. Zoomed region of Figure 4.

The maximum and minimum values of the calculated absorption coefficient, in other words the 'envelope' of the graphs in Figure 4, give a 95% confidence interval. It was found that the relationship between the transfer function and the absorption coefficient was quite complex and that a range of transfer function values were needed to adequately sample the 95% confidence interval. In this case the number of steps used to sample the transfer function is 41, as calculations with just 2 steps (the 95% confidence values) would not cover a large enough range of normal absorption coefficient, and gave inaccurate results.

The calculation process above gives the 95% confidence interval for normal absorption coefficient, but it does not provide any information about the kind of distribution that describes the data. In order to report the values in a standard form, if the assumption of a normally distributed absorption coefficient is made, then a corresponding mean and standard deviation can be calculated using Equations (4) and (5), given t_n.

When narrow band results are aggregated to one-third octave bands (Figure 6), the mean results appear to be similar to the one-third octave band results obtained using the simplistic method of calculation, but the confidence intervals are physically realisable.

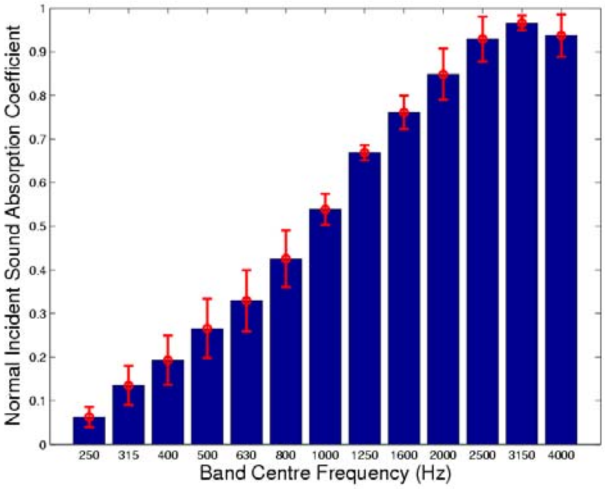

Figure 6. One-third octave band absorption coefficients mean and 95% confidence intervals estimated using the alternative approach.

Conclusions

An alternative method proposed for calculating the 95% confidence intervals for normal incidence sound absorption coefficient measurements using the two microphone technique is found to be robust and physically based, giving a realistic range of absorption coefficients covering 95% of the measurements. It is suggested that this method is adopted for future two microphone measurements where statistical information is required. Future work could remove the requirement for the final assumption of a normally distributed absorption coefficient by using Monte-Carlo or analytical techniques.

References

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