THE RADIATION EFFICIENCY OF FINITE SIZE FLAT PANELS

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Abstract

The radiation efficiency of an infinite flat panel which is radiating an infinite plane wave into an infinite half space can be shown to be equal to the inverse of the cosine of the angle between the direction of propagation of the plane wave and the normal to the panel. The fact that this radiation efficiency tends to infinity as the angle tends to 90° causes problems with simple theories of sound insulation. Sato has calculated numerical values of radiation efficiency for a finite size rectangular panel. This paper presents a simple analytic strip theory which agrees reasonably well with Sato's numerical calculations for a rectangular panel. This leads to the conclusion that it is mainly the length of the panel in the direction of radiation, rather than its width that is important in determining its radiation efficiency.

Nomenclature

a	Half length of source
c	Speed of sound in air
g	Cosine of angle of incidence
g_l	Cosine of limiting angle of incidence
I	Radiated sound intensity on one side
I_0	Reference radiated intensity on one side
k	Wave number in air
k_b	Wave number in panel
m	Constant
N	Number of sound sources
p	Sound pressure in air
p_{rms}	Root mean square sound pressure in air
q	Inverse of low frequency radiation efficiency
r	Radius of sphere or hemisphere
S	Surface area
t	Time
U	Perimeter
u	Particle velocity in air
v	Normal velocity of panel
v_{rms}	Root mean square normal velocity of panel
x	Variable of integration
y	Complement of angle of incidence
Z_c	Characteristic impedance of air
Z_{wf}	Fluid wave impedance of panel in air
z_{wf}	Normalised fluid wave impedance of panel
δ	Half total phase change at observer
θ	Angle of radiation relative to normal
λ	Wavelength in air
λ_b	Wavelength in panel
ρ_0	Ambient density of air
σ	Radiation efficiency
φ	Angle of incidence relative to normal
φ_l	Limiting angle of incidence relative to normal
ψ	Half change of phase across source
ω	Angular frequency

Introduction

If an infinite plane wave strikes a panel it forces a bending wave in the panel whose wavelength is greater than or equal to the wavelength of the incident wave in air. Because of this, the forced wave in the panel can radiate efficiently into air on its other side. In this paper we first derive the well known result that the radiation efficiency of an infinite panel is equal to the inverse of the cosine of the angle of incidence and transmission. This result obviously cannot be correct for a finite size panel because it goes to infinity at grazing incidence.

Gösele [1] derived the radiation efficiency for a finite panel. He also included panel wavelengths which are less than the wavelength of the sound in air for which the infinite panel model predicts zero radiation efficiency. He gave approximate formulae for certain ranges of parameters and graphed results of numerical calculations for three different sizes of panels.

Sato [2] gave the results of much more extensive numerical calculations in both tabular and graphical form for the forced wave case where the panel wavelength is longer than the wavelength in air. Sato also numerically calculated the radiation efficiency averaged over all possible directions of sound incidence.

Rindel [3] used Sato's numerical results for radiation efficiency in his theory of sound insulation as a function of angle of incidence. According to Novak [4], Lindblad [5] provided an approximate formula for the radiation efficiency at high frequencies based on Gösele's results. In [6], Lindbald also gave a simpler approximation which could be integrated over all angles of incidence. He also extended the integrated formula to low frequencies.

Rindel [7] presented a slightly more complicated version of Lindblad's more complicated formula, with constants which were selected to provide good agreement with Sato's tabulated radiation efficiencies. Rindel's formula also extended Lindbald's formula to low frequencies. This formula of Rindel is too complicated to be integrated easily by analytic means.

Ljunggren [8] repeated Sato's numerical calculations using a two dimensional model and obtained agreement "well within 0.5 dB" for both as a function of angle of incidence and averaged over all angles of incidence. Novak [9] has performed even more extensive three dimensional calculations than Sato.

The purpose of this paper is to derive an analytic approximation to Sato's numerical results using a simple two dimensional strip model. This analytic approximation has to be simple enough so that it can be integrated easily by analytic means over all angles of incidence for comparison with Sato's diffuse field results.

Infinite panels

Figure 1 shows an infinite plane sinusoidal sound wave incident on an infinite panel. The panel is coloured red and the direction of propagation of the infinite plane sinusoidal sound wave is shown by the green arrow. This direction of propagation is at an angle of θ to the normal to the panel. The normal to the panel is coloured mauve. The wave front maxima are coloured blue. They are separated by the wavelength λ of the infinite plane sinusoidal sound wave.

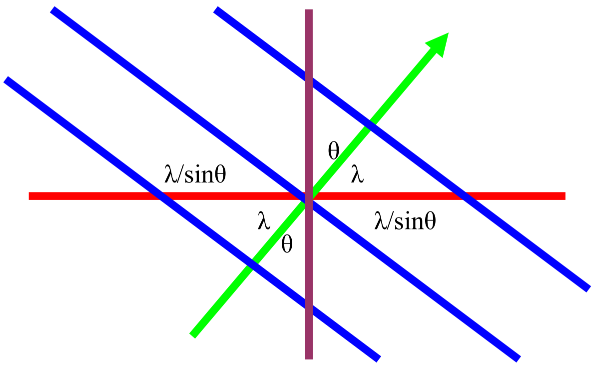

Figure 1. Infinite plane sinusoidal sound wave incident on an infinite panel

The distance between the wave front maxima measured along the panel is

$$\lambda_b = \frac{\lambda}{\sin\theta} \tag{1}$$

Thus λ_b is also the wavelength of the forced sinusoidal bending wave that the incident sinusoidal sound wave induces in the panel, because the wave front maxima of the forced bending wave must correspond with the wave front maxima of the incident wave.

Since the wave number is

$$k = \frac{2\pi}{\lambda} \tag{2}$$

$$k_b = k\sin\theta \tag{3}$$

The frequencies of the incident sound wave, the forced bending wave and the transmitted sound wave must all be equal. Since the speed of sound is the same on both sides of the panel, the wavelength of the transmitted sound wave must be equal to the wavelength λ of the incident wave. Because the wave front maxima of the transmitted wave must correspond to the wave front maxima of the forced bending wave, the transmitted sound wave must propagate at an angle of θ to the normal to the infinite panel.

If the particle velocity of the transmitted infinite plane sound wave is u, the component of the particle velocity normal to the panel is $u \cos \theta$. Continuity demands that this velocity is equal to the normal velocity v of the infinite panel. Continuity also dictates that the transmitted sound wave pressure and the pressure exerted by the panel to create the transmitted sound wave are the same pressure p.

If the density of the air is ρ_0 and the speed of sound in the air is c, then the characteristic impedance of air is

$$Z_c = \frac{p}{u} = \rho_0 c \tag{4}$$

The fluid wave impedance experienced by the panel on its radiating side is

$$Z_{wf} = \frac{p}{v} = \frac{p}{u\cos\theta} = \frac{Z_c}{\cos\theta} = \frac{\rho_0 c}{\cos\theta} \tag{5}$$

If the fluid wave impedance Z_{wf} is normalised by dividing by the characteristic impedance Z_c, the normalised fluid wave impedance is

$$z_{wf} = \frac{Z_{wf}}{Z_c} = \frac{1}{\cos\theta} \tag{6}$$

The sound power per unit area radiated by the panel on the transmitted side is

$$I = p_{rms} v_{rms}^* = \mathrm{Re}(Z_{wf}) v_{rms}^2 \tag{7}$$

The reference radiated power per unit area is

$$I_0 = Z_c v_{rms}^2 \tag{8}$$

The radiation efficiency of the panel is

$$\sigma = \frac{I}{I_0} = \frac{\mathrm{Re}(Z_{wf})}{Z_c} = \mathrm{Re}(z_{wf}) = \frac{1}{\cos\theta} \tag{9}$$

The fact that this radiation efficiency σ tends to infinity as the angle of incidence θ tends to 90° causes problems with simple theories of sound insulation. This result obviously cannot be correct for finite size panels.

Discrete and line sources

Figure 2 shows two point sound sources which are separated by a distance 2a which is shown as a red line. The two sound sources are sinusoidal with equal frequency and equal amplitude. An observer at a distance which is very large compared to the distance d which separates the sound sources will receive almost the same amplitude sound wave from each source. The lines from the two sound sources to the distant observer, which are shown in green, will be almost parallel.

The sound wave from source 1 has to travel an extra distance 2 a sin θ, where θ is the angle between the normal, shown in mauve, to the line joining the two sound sources and the parallel lines from the two sources to the distant observer. It will also be assumed that the phase of source 2 leads the phase of source 1 by 2 ψ. Thus at the distant observer, the phase of the sound from source 2 leads the phase of the sound from source 1 by

$$2\delta = 2\psi + 2ka\sin\theta \tag{10}$$

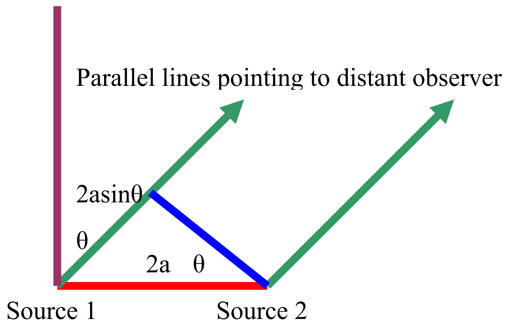

Figure 2. Two discrete sound sources

If ω is the angular frequency of the two point sound sources, at time t the amplitude of the sound at the distant observer is proportional to

$$\begin{aligned}&\frac{\sin(\omega t + 2\delta) + \sin(\omega t)}{2} = \cos\delta\sin(\omega t + \delta)\\ &= \frac{2\sin\delta\cos\delta}{2\sin\delta}\sin(\omega t + \delta) = \frac{\sin(2\delta)}{2\sin(\delta)}\sin(\omega t + \delta)\end{aligned} \tag{11}$$

Thus the amplitude of the sound at the distant observer is proportional to

$$\frac{\sin(2\delta)}{2\sin\delta} \tag{12}$$

Now assume that there are N sources in a line of length 2 a. Each source has an amplitude proportional to $1/N$, is a distance 2 a / (N - 1) from the previous source and leads the phase of the previous source by 2 ψ / (N – 1). At the distant observer, the phase of the sound from each source leads the phase of the sound from the previous source by

$$2\delta = \frac{2\psi + 2ka\sin\theta}{N-1} \tag{13}$$

The sound wave at the distant observer is proportional to

$$\begin{aligned}&\frac{1}{N}\sum_{n=1}^{N}\sin[\omega t + 2(n-1)\delta]\\ &= \frac{\sin(N\delta)}{N\sin(\delta)}\sin[\omega t + (N-1)\delta]\end{aligned} \tag{14}$$

The above summation has been performed using formula 1.341.1 on page 29 of Gradshteyn and Ryzhik [10].

If N is very large

$$N\delta \approx (N-1)\delta = \psi + ka\sin\theta \tag{15}$$

Thus

$$\delta = \frac{\psi + ka\sin\theta}{(N-1)} \ll 1 \tag{16}$$

and

$$\sin\delta = \delta \tag{17}$$

Thus the sound wave at the distant observer is proportional to

$$\begin{aligned}&\frac{\sin(N\delta)}{N\sin(\delta)}\sin[\omega t + (N-1)\delta]\\ &= \frac{\sin(\psi + ka\sin\theta)}{\psi + ka\sin\theta}\sin(\omega t + \psi + ka\sin\theta)\end{aligned} \tag{18}$$

This large N limit gives us the result for a continuous line source of constant source strength over a length of $2a$ and phase difference which varies linearly by a total amount of 2ψ over the length $2a$ of the continuous line source. The sound amplitude at a distant observer is proportional to

$$\frac{\sin(\psi + ka\sin\theta)}{\psi + ka\sin\theta} \tag{19}$$

If the phase difference ψ is due to a forced bending wave induced by a wave incident at an angle of φ

$$\psi = -k_b a = -ka\sin\varphi \tag{20}$$

In this case the sound amplitude at a distant observer is proportional to

$$\frac{\sin[ka(\sin\theta - \sin\varphi)]}{ka(\sin\theta - \sin\varphi)} \tag{21}$$

Infinite strips

We now consider an infinite strip of width $2a$ and ask how much power per unit length it radiates from one side when excited by an infinite plane sinusoidal wave incident at an angle of φ to the normal to the strip. The plane wave maxima planes are assumed to be parallel to the two parallel edges of the infinite strip. This is a two dimensional problem. We have to square the amplitude at each angle of radiation θ to obtain the power and sum over all angles of radiation by integrating the power over all angles of radiation θ from -π/2 rad to π/2 rad.

From integral 3.821.9 on page 446 of Gradshteyn and Ryzhik [10]

$$\int_0^\infty \frac{\sin^2(mx)}{x^2}dx = |m|\frac{\pi}{2} \tag{22}$$

Thus

$$\int_0^\infty \frac{\sin^2(mx)}{(mx)^2}dx = \frac{\pi}{2|m|} \tag{23}$$

and

$$\int_{-\infty}^\infty \frac{\sin^2(mx)}{(mx)^2}dx = \frac{\pi}{|m|} \tag{24}$$

We will make the following approximation

$$\begin{aligned}&\sin\theta - \sin\varphi = 2\sin\left(\frac{\theta-\varphi}{2}\right)\cos\left(\frac{\theta+\varphi}{2}\right)\\ &\approx (\theta-\varphi)\cos\varphi \quad \text{for } |\theta-\varphi| \ll 1\end{aligned} \tag{25}$$

We will also approximate by extending the limits of integration from -π/2 to π/2 to -∞ to ∞. We will examine the range of validity of this approximation later. With these approximations the total radiated sound power per unit length of strip is proportional to

$$\int_{-\infty}^{\infty}\frac{\sin^2[ka(\theta-\varphi)\cos\varphi]}{[ka(\theta-\varphi)\cos\varphi]^2}d\theta = \int_{-\infty}^{\infty}\frac{\sin^2(ka\theta\cos\varphi)}{(ka\theta\cos\varphi)^2}d\theta=\frac{\pi}{ka\cos\varphi} \quad (26)$$

This is the same 1/cosφ variability as in the case of the infinite panel since for the infinite panel, the transmitted angle θ is equal to the incident angle φ. Equation (26) is only proportional to the radiation efficiency of the infinite strip. Since the radiation efficiency of an infinite strip must equal the radiation efficiency of an infinite panel if ka is large enough, Equation (26) must be multiplied by ka/π to obtain the absolute value of radiation efficiency given by equation (9). This result has previously been obtained by Gösele [1].

We now have to investigate the range of validity of equation (26). The maximum value of

$$\frac{\sin^2[ka(\theta-\varphi)\cos\varphi]}{[ka(\theta-\varphi)\cos\varphi]^2} \quad (27)$$

is 1 when θ equals φ. Thus we will replace this function in equation (26) with a function which is equal to one when

$$|\theta-\varphi|\le\frac{\pi}{2ka\cos\varphi} \quad (28)$$

and is zero elsewhere. This function gives the same value for the integral. For this replacement function the change to the limits of integration is only valid if the nonzero part of the replacement function lies between -$\pi/2$ to $\pi/2$. This means that

$$\frac{\pi}{2}-|\varphi|\ge\frac{\pi}{2ka\cos\varphi} \quad (29)$$

For $|\varphi|$ close to $\pi/2$

$$\frac{\pi}{2}-|\varphi|\approx\cos\varphi \quad (30)$$

Thus equation (29) becomes

$$\cos\varphi\ge\frac{\pi}{2ka\cos\varphi} \quad (31)$$

or

$$\cos\varphi\ge\sqrt{\frac{\pi}{2ka}} \quad (32)$$

or

$$|\varphi|\le\arccos\sqrt{\frac{\pi}{2ka}} \quad (33)$$

Thus equation (26) is only valid in the range given by equation (33). At the two angles of incidence φ_l given by the equal sign in equation (33), the total radiated sound power per unit length of strip is proportional to

$$\frac{\pi}{ka\cos\varphi_l}=\frac{\pi}{ka}\sqrt{\frac{2ka}{\pi}}=\sqrt{\frac{2\pi}{ka}} \quad (34)$$

Since the maximum value of the function in equation (27) is one, the maximum value of the integral before we extended the limits is $\pi/2 - (-\pi/2) = \pi$. Also cos φ is in the range from zero to one for all values of φ in the range from -$\pi/2$ to $\pi/2$. Thus we have

$$\frac{\pi}{ka}\le\frac{\pi}{ka\cos\varphi}\le\pi \quad (35)$$

This means that our approximations can only be valid if ka is greater than or equal to one.

It is also possible to approximate the integral if $|\varphi| = \pi/2$. Because of symmetry in the equations we only need to consider the case $\varphi = \pi/2$. We have

$$\sin(\theta)-\sin(\varphi)=\cos\left(\frac{\pi}{2}-\theta\right)-1 \quad (36)$$

If $\pi/2 - \theta$ is small equation (36) becomes

$$1-\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)^2-1=-\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)^2 \quad (37)$$

Put

$$y=\frac{\pi}{2}-\theta \quad (38)$$

then

$$ka[\sin(\theta)-\sin(\varphi)]=-kay^2/2 \quad (39)$$

The integral becomes

$$\int_0^{\infty}\frac{\sin^2(kay^2/2)}{(kay^2/2)^2}dy \quad (40)$$

The $\theta = \pi/2$ limit has become y = 0. The θ = - $\pi/2$ limit has become $y = \pi$ and been extended to $y = \infty$.

Integral number 3.852.3 on page 464 of Gradshteyn and Ryzhik [10] is

$$\int_0^{\infty}\frac{\sin^2(m^2x^2)}{x^4}dx=\frac{2\sqrt{\pi}}{3}m^3 \text{ for } m\ge0 \quad (41)$$

Using equation (41), equation (40) becomes

$$\left(\frac{2}{ka}\right)^2\frac{2\sqrt{\pi}}{3}\left(\frac{ka}{2}\right)^{3/2}=\frac{2}{3}\sqrt{\frac{2\pi}{ka}} \quad (42)$$

Like Equation (26), Equation (42) must be multiplied by ka/π to obtain the absolute value of the radiation efficiency. This result has previously been derived by Gösele [1]. It should be noted that it is 2/3 of the maximum value derived in equation (34) for

$$\cos\varphi_l=\sqrt{\frac{\pi}{2ka}} \quad (43)$$

Finite size square panels

To extend our results to values of ka less than one, we now assume that we are dealing with a finite size square panel with sides of length $2a$. Since we are only interested in the power that is radiated we only have consider the real part of the normalised fluid wave impedance z_{wf}. For a symmetrically pulsating sphere of radius r, the real part of the normalised fluid wave impedance for $kr << 1$ is k^2r^2. By symmetry this result also applies to a pulsating hemisphere whose centre is on an infinite rigid plane. For sources whose size is small compared to the wavelength of sound, it is expected that their sound radiation will depend only on their volume velocities. Thus the result for the pulsating sphere will also apply to a square panel set in an infinite rigid plane baffle providing the area of the square panel is equal to the surface area of the hemisphere. Thus $2\pi r^2 = 4a^2$ and the radiation efficiency of the square panel is

$$\sigma = \mathrm{Re}(z_{wf}) = \frac{\mathrm{Re}(Z_{wf})}{Z_c} = k^2r^2 = \frac{2}{\pi}k^2a^2 \qquad (44)$$

Combining this result with our infinite panel and infinite strip results gives a radiation efficiency of

$$\sigma(\varphi) = \begin{cases} \dfrac{1}{\dfrac{\pi}{2k^2a^2} + \cos\varphi} & \text{if } |\varphi| \le \varphi_l \\ \dfrac{1}{\dfrac{\pi}{2k^2a^2} + \dfrac{3\cos\varphi_l - \cos\varphi}{2}} & \text{if } \varphi_l < |\varphi| \le \dfrac{\pi}{2} \end{cases} \qquad (45)$$

In Equation (45) the result has been interpolated linearly in cos φ between the result at $|\varphi| = \varphi_l$ and the result at $|\varphi| = \pi/2$.

Table 1. Difference in decibels between the radiation efficiency given by Equation (45) and Sato's [2] numerically calculated radiation efficiency.

ka	0°	15°	30°	45°	60°	75°	90°
0.5	-0.3	-0.3	-0.4	-0.3	-0.3	-0.2	-0.3
0.75	-0.8	-0.7	-0.7	-0.6	-0.5	-0.5	-0.6
1	-1.1	-1.1	-0.9	-0.7	-0.6	-0.6	-0.7
1.5	-1.7	-1.5	-1.4	-1.0	-0.7	-0.6	-0.7
2	-2.8	-2.6	-0.9	-0.6	-0.4	-0.2	-0.4
3	-1.2	-1.4	-1.3	0.0	0.2	0.2	-0.1
4	-0.5	-0.7	-1.0	-0.6	0.4	0.3	0.0
6	-0.4	-0.4	-0.4	-0.6	0.8	0.6	0.2
8	-0.3	-0.3	-0.1	-0.4	0.2	0.7	0.2
12	-0.1	-0.1	-0.1	0.0	-0.3	0.8	0.3
16	-0.1	-0.1	0.0	-0.1	-0.3	0.9	0.2
24	0.0	-0.1	0.0	0.0	0.1	1.1	0.2
32	0.0	-0.1	0.0	0.0	0.0	0.4	0.2
48	0.0	-0.1	0.0	0.0	0.0	-0.2	0.2
64	0.0	-0.1	0.0	0.0	0.0	-0.2	0.1

The radiation efficiency averaged over all angles of incidence φ is

$$\langle\sigma\rangle = \int_0^{\pi/2} \sigma(\varphi)\sin\varphi\, d\varphi \qquad (46)$$

The sin φ occurs in the integral because there is more solid angle for sound to be incident from the closer φ is to $\pi/2$. To evaluate this integral, the following substitutions are made.

$$q = \frac{\pi}{2k^2a^2} \qquad (47)$$

$$g_l = \cos\varphi_l = \sqrt{\frac{\pi}{2ka}} \qquad (48)$$

$$g = \cos\varphi \qquad (49)$$

Hence

$$dg = -\sin\varphi\, d\varphi \qquad (50)$$

Equation 46 becomes

$$\begin{aligned} \langle\sigma\rangle &= \int_{g_l}^{1} \frac{dg}{q+g} + 2\int_0^{g_l} \frac{dg}{2q+3g_l-g} \\ &= \ln\left(\frac{q+1}{q+g_l}\right) + 2\ln\left(\frac{2q+3g_l}{2q+2g_l}\right) \end{aligned} \qquad (51)$$

Table 2. Difference in decibels between various diffuse field radiation efficiency approximations and Sato's [2] numerically calculated diffuse field radiation efficiency.

ka	D	L1	L2	R	S
0.5	-0.27	0.46	-2.41	-1.50	-0.72
0.75	-0.53	0.81	0.06	0.34	0.31
1	-0.70	1.09	0.10	0.29	0.19
1.5	-0.86	1.15	0.01	0.13	0.03
2	-0.52	0.84	0.04	0.14	0.05
3	-0.10	0.07	0.04	0.12	0.04
4	0.14	0.06	0.09	0.16	0.08
6	0.27	0.05	0.06	0.12	0.05
8	0.32	0.05	0.06	0.11	0.05
12	0.35	0.05	0.05	0.10	0.04
16	0.32	0.02	0.02	0.06	0.01
24	0.30	0.01	0.01	0.05	0.00
32	0.27	-0.01	-0.01	0.02	-0.02
48	0.24	-0.03	-0.03	0.00	-0.04
64	0.21	-0.05	-0.05	-0.02	-0.05

Comparison with Calculations

Table 1 shows that Equation (45) is always between -2.8 dB and +1.1 dB of Sato's [2] numerical results. The biggest errors result from the combination of the high frequency and low frequency results in the region of *ka* = 2. This is why most other authors have not extended their approximations to low frequencies. Rindel's approximation [7] differs from Sato's tabulated results by between -1.4 dB and +0.9 dB, but is too complicated be easily analytically integrated.

Lindblad [6] only applied a low frequency correction to his integrated approximation. Applying the same low frequency correction to Lindblad's unintegrated approximations gives a range of -1.3 dB to +1.8 dB relative to Sato's tabulated numerical results for Lindbald's more complicated approximation which cannot be easily analytically integrated. Lindblad's simpler approximation which can be analytically integrated gives a range of -0.6 dB to +1.8 dB relative to Sato's numerical results. Novak [4] used Lindblad's more complicated formula with a combining power of ten rather than the combining power of four used by Lindblad. Applying Lindbald's low frequency correction, Novak's approximation agreed with Sato's numerical results within range of -0.9 dB and +1.8 dB. Again Novak's approximation cannot be easily integrated analytically over all angles of incidence.

Table 2 shows that Equation (51) for the diffuse field incidence (D), which is obtained by averaging over all possible angles of incidence, is always between -0.86 dB and +0.35 dB of Sato's [2] numerical results. L1 in Table 2 is Lindblad's diffuse field result from his simplified approximation with Lindblad's low frequency correction. L1 agrees with Sato's numerically calculated diffuse field results within -0.05 dB and +1.15 dB. It is interesting to note in Table 2 that L2, which is L1 without the low frequency correction, agrees with Sato's numerical calculations within -0.05 dB and +0.10 db for $ka > 0.5$. At $ka = 0.5$, the lack of the low frequency correction makes the difference -2.41 dB. The equation for L2 is

$$\langle \sigma \rangle = 1 + \ln\left(\sqrt{\frac{ka}{\pi}}\right) \tag{52}$$

Setting the low frequency correction q in Equation (51) to zero produces Equation (52) with the 1 changed to 1.16.

Rindel [11] gives a diffuse field radiation efficiency approximation R which is very similar to L2.

$$\langle \sigma \rangle = \frac{1}{2}\left(0.2 + \ln 2ka\right) \tag{53}$$

Rindel says that this approximation is useful for $ka > 0.5$. Table 2 shows that it agrees with Sato's tabulated numerical results within -0.02 dB and +0.34 dB for $ka > 0.5$. At $ka = 0.5$, the lack of the low frequency correction makes the difference -1.50 dB. Setting the low frequency correction q in Equation (51) to zero produces Equation (53) with the 0.2 changed to 0.239.

Sewell's work [12] can be interpreted as producing a similar formula with a low frequency correction.

$$\langle \sigma \rangle = \frac{1}{2}\left(0.160 + \ln 2ka + \frac{1}{16\pi k^2 a^2}\right) \tag{54}$$

Table 2 shows that this formula S agrees with Sato's tabulated numerical results within -0.72 dB and +0.31 dB. Sewell's work also gives a correction for non-square rectangular panels.

For a specific incidence direction *2a* should be set equal to a typical length of the panel in that direction. For averages over all azimuthal angles *2a* should be set equal to $\frac{\pi S}{U}$ [8, 9], $\frac{4S}{U}$ [7, 11] or $\sqrt{S}$ [12] where S is the area and U is the perimeter of the panel.

Conclusions

The two dimensional strip model analytic approximation derived in this paper gives reasonable agreement with three dimensional numerical calculations. This agrees with Ljunggren [8] whose two dimensional numerical calculations agree within ±0.5 dB of the three dimensional calculations of Sato [2] and Novak [9]. It also agrees with the experimental measurements of Roberts [13] which show that the directivity of a rectangle depends strongly on its length in the direction of measurement but only weakly on its width at right angles to the direction of measurement.

Thus we can conclude that the radiation efficiency of a forced wave on a panel is mainly determined by the ratio of its length in the direction of measurement to the wavelength of the sound in air and the angle of incidence of the forcing wave.

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