ROBUST HIGH RESOLUTION DOMINANT MODE REJECTION (DMR) BEAMFORMER FOR PASSIVE SONAR

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Abstract

Resolving the bearings of acoustic sources is one of the most important objectives of sonar arrays. In this paper, we present a robust high bearing resolution adaptive beamforming (ABF) algorithm. The algorithm is based on two existing ABF methods both being extensions of the well-known minimum variance distortionless response (MVDR) method. One is dominant mode rejection with eigenvector-beam association and excision (DMR/EBAE) beamforming that gives high bearing resolution. The other is robust Capon beamforming (RCB) that provides excellent robustness. Applying the algorithm to sonar data from sea trials shows that the proposed algorithm exhibits excellent robustness; it performs well even in the presence of failed sensors in the sonar array. It also provides improved performance in terms of interference rejection and signal separation compared to other existing ABF algorithms.

Introduction

Beamforming of sensor arrays is a fundamental operation in sonar, radar, seismology and telecommunications. One of the most important objectives of beamforming in passive sonar is to resolve the bearings of acoustic sources [1]. To improve the bearing resolution of beamforming, various methods have been proposed in the past [2]. Adaptive beamforming (ABF) methods are known to have superior bearing resolution capabilities compared to conventional non-adaptive beamforming (CBF). However, this resolution advantage over CBF can degrade in the presence of signal mismatch. The eigenvector-based ABF has been proved to be an effective way to maintain the resolution advantage of ABF [3, 4]. It allows individual eigenvectors to be removed in the calculation of weighting vectors of the beamformer. This provides a protection against ‘self-nulling’ that causes the resolution to be degraded. In this paper, we present an ABF algorithm that is very similar to the algorithm proposed in [4] but is more robust.

In the next section, we first give a brief review of the basic MVDR (Minimum Variance Distortionless Response) beamforming algorithm, and the DMR/EBAE (Dominant Mode Rejection/Eigenvector-Beam Association and Excision) algorithm proposed by Kogon [4]. Then, we present a robust high resolution DMR algorithm. The algorithm is based on the DMR/EBAE algorithm and the RCB (Robust Capon Beamforming) algorithm proposed by Li *etc* [5]. In the section after, we compare the performance of the robust high resolution DMR algorithm to those of the basic MVDR and the DMR/EBAE algorithms by using experimental data from sea trials. Finally, we draw conclusions in the last section.

Background

Basic MVDR Beamforming

The basic MVDR beamforming algorithm is the starting point for many ABF algorithms and serves as a basis for the algorithm presented in this section.

Let $\mathbf{R}$ be the $M\times M$ cross-spectral matrix of the array output data and $\mathbf{v}$ the $M\times 1$ signal response vector obtained from the signal model. The MVDR beamformer is the solution of the following linearly constrained quadratic problem:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}\mathbf{w} \quad \text{subject to } \mathbf{w}^H \mathbf{v} = 1 , \tag{1}$$

where $\mathbf{w}$ is $M\times 1$ weighting vector of the beamformer. The solution is well known [2]:

$$\mathbf{w_0} = \frac{\mathbf{R}^{-1}\mathbf{v}}{\mathbf{v}^H \mathbf{R}^{-1}\mathbf{v}} . \tag{2}$$

DMR/EBAE Beamforming

A number of techniques fall under the general classification of DMR method. The DMR/EBAE algorithm was proposed by Kogon [4]. It provides a higher spatial resolution than the basic MVDR algorithm in the presence of signal mismatch.

The basic idea of DMR method is to estimate only the large eigenvalues and their corresponding eigenvectors of $\mathbf{R}$, and to use them to null the strong interferences that they represent. The advantage is that less averaging (fewer snapshots) is required. This permits rapid convergence.

The cross-spectral matrix $\mathbf{R}$ can be decomposed into the following terms

$$\mathbf{R} = \sum_{i=1}^{D} \lambda_i \mathbf{\Phi}_i \mathbf{\Phi}_i^H + \sum_{i=D+1}^{M} \lambda_i \mathbf{\Phi}_i \mathbf{\Phi}_i^H , \tag{3}$$

where λ_i are the eigenvalues of $\mathbf{R}$, $\mathbf{\Phi}_i$ the corresponding eigenvectors and D is the number of the larger eigenvalues.

By replacing the (M-D) smallest eigenvalues in (3) with their average,

$$\alpha = \frac{1}{M-D}\sum_{i=D+1}^{M}\lambda_i \ , \qquad (4)$$

we form a modified spectral matrix

$$\hat{\mathbf{R}} = \sum_{i=1}^{D}\lambda_i \mathbf{\Phi}_i \mathbf{\Phi}_i^H + \alpha \sum_{i=D+1}^{M}\mathbf{\Phi}_i \mathbf{\Phi}_i^H \ . \qquad (5)$$

The inverse of $\hat{\mathbf{R}}$ is

$$\hat{\mathbf{R}}^{-1} = \sum_{i=1}^{D}\frac{1}{\lambda_i}\mathbf{\Phi}_i \mathbf{\Phi}_i^H + \frac{1}{\alpha}\sum_{i=D+1}^{M}\mathbf{\Phi}_i \mathbf{\Phi}_i^H \ . \qquad (6)$$

By replacing $\mathbf{R}^{-1}$ in (2) with $\hat{\mathbf{R}}^{-1}$, one has the basic DMR algorithm.

It is known that for the basic DMR and MVDR algorithms, 'self-nulling' will occur in the presence of signal mismatch. This degrades the resolution of the beamformer. To overcome this problem, Kogon proposed the EBAE scheme [4]. The basic idea of the scheme is to identify the offending eigenvector that causes self-nulling and remove it from (6) when computing $\hat{\mathbf{R}}^{-1}$. The offending eigenvector is identified as the one that gives the largest beam response computed by the following formula:

$$\left\|\mathbf{v}^H \mathbf{\Phi}_i\right\|^2 \qquad \text{for } i = 1 \ to \ D \qquad (7)$$

Robust High Resolution DMR Beamforming

The DMR/EBAE algorithm is still sensitive to large signal mismatch (eg., failed sensors in an array). To overcome the sensitivity to large signal mismatch in basic MVDR beamforming, Li *etc* proposed the robust Capon beamforming (RCB) algorithm [5]. In their algorithm, they converted the quadratic problem in (1) into the new quadratic problem as follows:

$$\min_{\mathbf{v}_a} \mathbf{v}_a^H \mathbf{R}^{-1}\mathbf{v}_a \qquad \text{subject to} \left\|\mathbf{v}_a - \mathbf{v}\right\|^2 \le \varepsilon \qquad (8)$$

where v_a is the actual signal response vector (which is unknown and to be estimated) and ε is a positive number proportional to signal mismatch. By replacing $\mathbf{v}$ in (2) with $\mathbf{v}_a$, which is the solution to (8), they derived the RCB algorithm.

The robust high resolution DMR approach is the combination of the DMR/EBAE algorithm and the RCB algorithm. It consists of the following steps.

Step 1. Compute the eigendecomposition of $\mathbf{R}$.

Step 2. Use the EBAE scheme to compute $\hat{\mathbf{R}}^{-1}$ [4].

Step 3. Solve the quadratic problem in (8) for $\mathbf{v}_a$ [5].

Step 4. Compute $\mathbf{w}_o$:

$$\mathbf{w_o} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{v}_a}{\mathbf{v}_a^H \hat{\mathbf{R}}^{-1}\mathbf{v}_a} \ . \qquad (9)$$

Step 5. Compute the output of the beamformer:

$$P = \mathbf{w}_o^H \mathbf{R}\mathbf{w}_o \ . \qquad (10)$$

Experiment

In this section, the performance of the robust high resolution DMR algorithm is compared to those of the basic MVDR and the DMR/EBAE algorithms by using experimental data collected by a passive sonar array.

Data Set with a Failed Sensor

In this data set, one of the sensors in a section of the array failed and its output was made zero. The data set contained two contacts. Figure 1 shows the outputs of the robust high resolution DMR, the basic MVDR and the DMR/EBAE algorithms at the design frequency of the array. We also include the output of the conventional beamformer (CBF) in the figure as a reference. The basic MVDR algorithm fails completely; no meaningful information presents in its output. The DMR/EBAE algorithm also fails; four peaks show in its output instead of two peaks. The noise floor is also raised (compared to the CBF). The robust high resolution DMR algorithm performs very well; it correctly detects the two contacts with the considerably lowered noise floor (high deflection SNR) and very sharp peaks (high resolution).

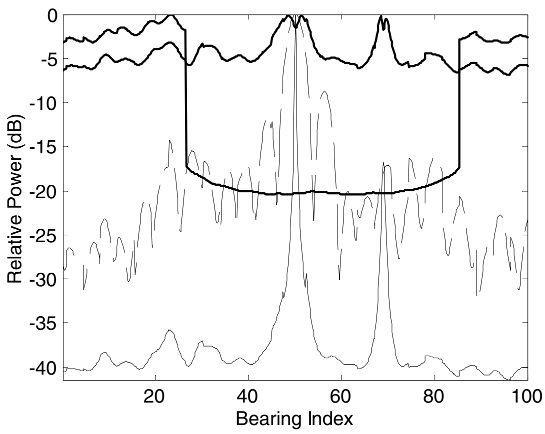

Figure 1. Beamformer outputs of the robust high resolution DMR (solid line), the DMR/EBAE (dot line), the basic MVDR (dash-dot line) and the CBF (broken line).

Data Set with Close-Spaced Multiple Contacts

There are four close-spaced contacts contained in this data set. Figure 2 shows the outputs of the robust high resolution DMR, the basic MVDR and the DMR/EBAE algorithms at the design frequency of the array. The basic MVDR algorithm is only capable of detecting three contacts. The second contact from right is not present in its output due to 'self-nulling'. Thanks to the EBAE scheme, the 'self-nulling' effect in the output of the DMR/EBAE algorithm is greatly reduced. As a result, the DMR/EBAE algorithm is capable of detecting all four contacts with increased deflection SNR. The performance of the robust high resolution DMR algorithm is better than that of the DMR/EBAE algorithm with considerably improved reflection SNR (more than 5 dB) and higher resolution.

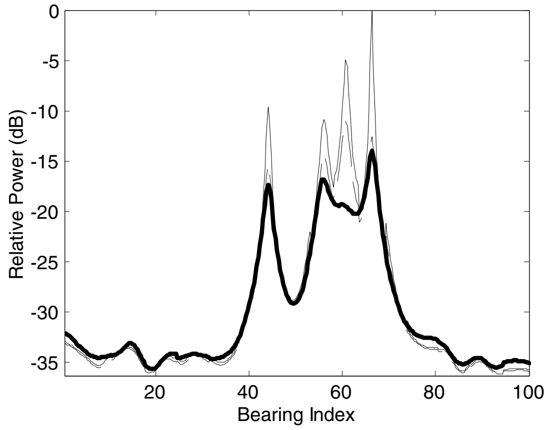

Figure 2. Beamformer outputs of the robust high resolution DMR (solid line), the DMR/EBAE (broken line), and the basic MVDR (dot line).

Sonar Application

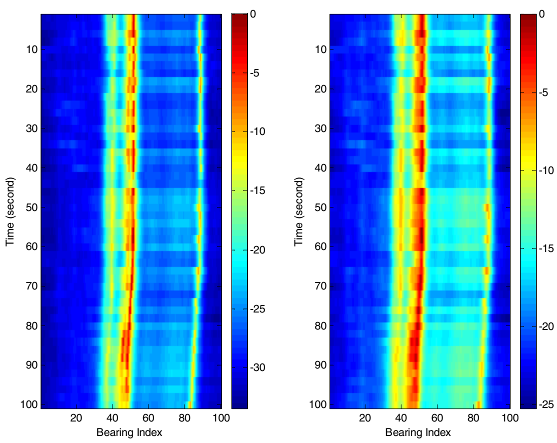

Figure 3. BTR for the robust high resolution DMR (left) and the basic MVDR (right).

In this section, we compare the bearing-time records (BTR) of the robust high resolution DMR algorithm to those of the basic MVDR algorithm. BTR display is widely used in sonar applications. The BTRs showed here are generated by using a broadband detection algorithm. Essentially, a series of narrowband beamforming is conducted and their outputs are integrated over frequency to form a broadband output. Figure 3 shows the BTR display of the sonar data in a period of 100 seconds. The plot on the left is the BTR of the robust high resolution DMR algorithm and the one on the right is of the basic MVDR algorithm. It can be seen that the resolution of the robust high resolution DMR algorithm is indeed higher than that of the basic MVDR algorithm. Especially, the basic MVDR algorithm fails to separate the group of very close-spaced contacts at about bearing index 50. In contrast, two contacts can be identified in the display of the robust high resolution DMR algorithm.

Conclusions

In this paper, we have presented a robust high resolution DMR beamforming algorithm. The algorithm is based on the DMR/EBAE method proposed by Kogon [4] and the RCB method proposed by Li *etc* [5]. Applying the algorithm to sonar data of sea trials shows that the proposed algorithm exhibits excellent robustness; it performs well even in the presence of failed sensors in the sonar array. Its spatial (bearing) resolution is at least as high as that of the DMR/EBAE method and is higher than that of the basic MVDR method.

References

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