SOME APPLICATIONS OF NUMERICAL ACOUSTICS

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ABSTRACT: The advantages and disadvantages of finite element and boundary element methods and geometrical acoustics methods are discussed. Applications of these methods to solving a range of vibro-acoustics problems and architectural acoustic problems are illustrated with practical examples including structural radiation, noise barriers and acoustic quality of rooms. In all these examples, comparisons of predictions are made with measurements to illustrate the accuracies, limitations and usefulness of these numerical methods.

1. INTRODUCTION

In acoustics, the propagation of sound is governed by the Helmholtz equation, first given by Euler in 1759 and then by Helmholtz in 1860. However, it was difficult to obtain analytical solutions with complex geometries and/or complex boundary conditions. With the advent of powerful desktop computers/workstations, numerical techniques based on geometrical acoustics (primarily ray tracing and mirror-image source methods for high frequencies) have been applied to problems in room acoustics and environmental acoustics. While commercial finite element software has been available for over three decades for engineering applications such as stress analysis, it is only recently that commercial software utilising advanced numerical techniques such as finite element and boundary element methods (primarily more suitable for low frequencies) has been made available. It has often been claimed that the most effective noise control method is the control of the source through engineering means but this is often very difficult to achieve unless prior considerations have been given to the design of a product. With the use of computational methods for acoustics, it is now possible to incorporate 'numerical' acoustic analysis in the design process so that noise radiation can be analysed even before prototypes are built and innovative engineering techniques in reducing noise radiation may be examined. However, there is generally a lack of verification of computer models against measurements except for simple geometries [1]. The objectives of this paper are to describe our experience in the use of boundary element and geometrical acoustics methods for applications in acoustics. Comparisons between predictions and measurements are made.

2. GOVERNING EQUATION AND BOUNDARY CONDITIONS

The propagation of sound waves in a medium is governed by the familiar wave equation:

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \qquad (1)$$

where c is the speed of propagation of sound waves; p is pressure; t is time; and ∇^2 is the Laplace operator.

By assuming a steady state harmonic motion of the form $p(x,y,z,t)=p(x,y,z)e^{j\omega t}$, ($j$ is the imaginary number $\sqrt{-1}$), equation (1) can be reduced to the Helmholtz equation:

$$\nabla^2 p + k^2 p = 0 \qquad (2)$$

where k is the wavenumber = ω/c and ω is the circular frequency in radians/sec.

Equation (2) can be solved by imposing appropriate boundary conditions on boundary surfaces which involve prescribing usually (a) the surface pressure (p_s); (b) the normal surface velocity(v_n); or (c) the normal surface admittance(A) or impedance as follows:

$$p = p_s, \qquad \frac{\partial p}{\partial n} = -j\rho\omega v_n, \qquad \frac{\partial p}{\partial n} = -j\rho\omega A p \qquad (3)$$

where n is the coordinate normal to the surface.

Furthermore, for external radiation problems, the acoustic field vanishes at points farther than $c(t-t_o)$ because a wave disturbance initiated at time (t_o) would not have reached that distance in the time (t) of interest. This condition is known as the Sommerfeld radiation condition [2] and may be expressed in spherical coordinates as

$$\lim_{r\to\infty} r^{\alpha}\left[\frac{\partial p}{\partial r} + jkp\right] \to 0 \qquad (4)$$

where r is the distance from the surface of source excitation and α is 1/2 for 2-D problems and 1 for 3-D problems.

The boundary-value problem as described by equations (2)-(4) is difficult to solve analytically except for very simple geometries and boundary conditions. Approximate analytical solutions can be obtained for high and low frequencies using perturbation methods [3]. Consequently, numerical methods have to be sought for general problems.

3. NUMERICAL METHODS

3.1 Finite element/Boundary element Methods

A good description of the implementation of finite element and boundary element methods (FEM/BEM) to solve the acoustic wave equation is given in [4, 5]. The basic differences between FEM and BEM are that in FEM, the whole solution domain has to be discretised while in BEM, only the boundary surface of the model has to be discretised as shown in Figure 1. Although FEM performs quite well for interior radiation problems, it is not so suitable for solving exterior radiation problems which would require an 'infinite' expansion of the finite element mesh. Nevertheless, 'infinite' elements and 'wave envelope' elements are being developed to solve exterior radiation problems [6]. Users of BEM must be aware that for exterior radiation problems, the solution obtained may not be unique at frequencies that correspond to the interior cavity resonant frequencies but this can be overcome by using special procedures as described in [5].

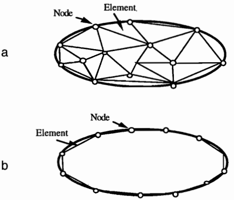


Figure 1 Discretisation of the solution domain.
(a) Finite element (b) Boundary element

3.2 Methods based on Geometrical acoustics

Solving equations (1)-(4) at high frequencies using FEM/BEM for large spaces would require an enormous amount of computer memory and disk capacity and can be prohibitively time consuming. At high frequencies, where the dimensions of the room are large compared with the wavelength, sound can be considered to behave as rays and the principles of specular reflection (ie the angle of incidence equals the angle of reflection) can be applied. There are now numerous commercially available software programs for doing such calculations. The algorithms used for such computer models are normally based on either the mirror image source method, the ray tracing method or the beam method [7]. Comparisons between results obtained by 14 different programs and measurements indicate that there are large differences between predictions by various algorithms [8]. Areas identified for further improvement include diffuse reflections.

4. EXAMPLES

All calculations reported here were made on a SUN SPARC20 workstation. Calculations using BEM were made using SYSNOISE version 5.3A while those using geometrical acoustics were made using RAYNOISE version 2.1

4.1 BEM

Radiation from an electric motor

It has been shown that the sound power level radiated from simple structures such as plates [6,9] and circular cylindrical shells [10] can be predicted with reasonable accuracy using BEM. In this example, the sound radiation efficiency of a 2.2 kW induction motor subjected to random mechanical excitation applied to a point on the casing has been determined experimentally and using BEM.

In the experiment, the sound power spectrum due to the mechanical excitation was measured in an anechoic room using a two-microphone sound intensity probe while the vibration spectra at 130 points distributed over the motor casing and the base plate were measured using an accelerometer. The sound radiation efficiency was then determined from these measurements.

In the numerical calculations, the motor structure was modelled using two concentric cylindrical shells, one for the casing and the other for the stator. As shown in Figure 2, the motor casing was modelled using 1128 quadrilateral shell elements and the stator was modelled using 720 solid elements. In this structural model of 3423 elements analysed using a commercial finite element code ANSYS version 5.4, the rotor has not been included because its contribution to the noise radiation is only significant for frequencies below 500 Hz [11]. Various factors affecting the accuracy of modelling a motor structure using finite elements have been discussed by Wang and Lai [11]. By using the results of the vibration response of the structural model as an input to the acoustic boundary element model, the sound radiation efficiency from the motor structure can be calculated.

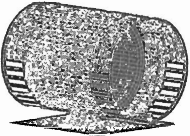


Figure 2 Perspective view of the structural model of an electric motor (with end shields removed).

As shown in Figure 3, there is reasonable agreement between the calculated and measured sound radiation efficiencies. The discrepancies at low frequencies can be attributed to the omission of the rotor in the model. Nevertheless, this example shows that such a model can be used to examine the effects of geometrical parameters such as thickness, stiffness, ribs, etc. on the sound radiation from a motor structure.

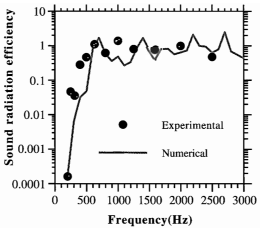


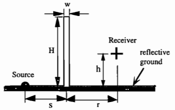
Figure 3 Sound radiation efficiency of a 2.2 kW induction motor.

Noise barriers

While it is rather routine to predict analytically or empirically the insertion loss of simple barriers such as shown in Figure 4(a), such prediction for innovative design is by no means trivial. The noise reducer (as shown in Figure 4(b)) is a cylindrical absorptive structure generally fitted to the top of a flat barrier. According to its manufacturer, Nitto Boseki Co., Ltd. in Japan, the noise reducer increases the effective height of the barrier by at least double the diameter of the noise reducer. It presents, therefore, a good opportunity to use the BEM calculations to assess the manufacturer's claim.

In this example, the ground is reflective, the source is a cylindrical line source located at 15 m from the barrier and the receiver is located at various distances (15 m, 25 m and 50 m) from the barrier. In order to eliminate interference effects due to ground reflections, calculations were made for both the source and the receiver at ground level.

a



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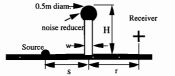
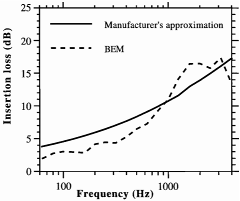


Figure 4 Schematic of barrier configurations.
(a) Flat barrier (b) Barrier fitted with Noise Reducer

a



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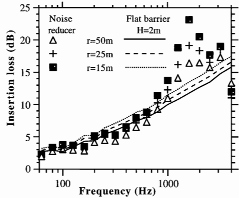

Figure 5 Insertion Loss.
(a) Noise reducer (b) Noise reducer and a flat barrier

Figure 5(a) shows that there are discrepancies between the BEM predicted insertion loss and the manufacturer's simplified approximation perhaps due to the assumed flow resistivity of the absorption material used in the noise reducer and the simplified cylindrical geometry used for modelling the noise reducer. Nevertheless, the predicted insertion loss is quite acceptable. More importantly, it allows the assessment of the effectiveness of a noise reducer fitted to a flat barrier with a total height of 1.5 m compared with that of a 2m high reflective flat barrier. Figure 5(b) shows that at frequencies above 800 Hz, the noise reducer has increased significantly the effective height of the barrier. The effectiveness of the noise reducer at 2000 Hz is illustrated by the sound pressure fields shown in Figure 6.

4.2 Geometrical Acoustics

Figure 7 shows a room with a volume of approx 3,700 m^3 modelled using geometrical acoustics. Measurements were made in octave frequency bands using a white noise sound source. The agreement between the predicted and measured early decay time (EDT) is generally within 0.2 sec over the important speech frequency range (Figure 8(a)) for two

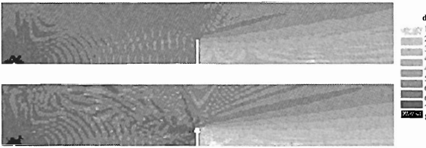

Figure 6
Sound pressure field at 2000 Hz

(a) reflective flat barrier with H=2 m

(b) reflective flat barrier with noise reducer with H=1.5 m

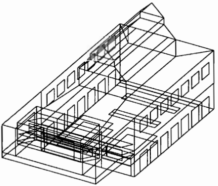

Figure 7 Room model.

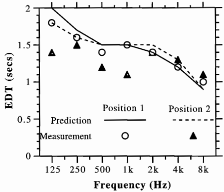

Figure 8(a) Comparisons of EDT.

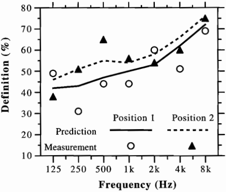

Figure 8(b) Comparisons of Definition.

different positions in the room. One indicator of speech clarity is 'Definition', D50, which is the ratio of the sound energy received in the first 50 ms to the total sound energy. A value for D50 of about 65% is equivalent to almost 95% speech intelligibility. Comparisons between the predicted and measured D50 in Figure 8(b) show agreement, generally to within 10-20%. It is important to point out that in this type of modelling, the source has to be modelled as accurately as possible. As seen in Figure 9, an omnidirectional source yields substantially different results from those of a directional source used in the experiments. It can be seen from Figure 10(a) that the predicted D50 at 1 kHz in the seating areas range from as low as 35% to around 60%. Numerical modelling allows the effects of any proposed changes on the acoustics quality to be assessed. Figure 10(b) shows that by implementing the proposed architectural modifications to the room, the predicted D50 at 1 kHz has been significantly increased to around 60-70%.

5. CONCLUSIONS

Applications of numerical acoustics methods (primarily boundary element and geometrical acoustics methods) are illustrated by practical examples from an electric motor, noise barriers and architectural acoustics. Results show that although there are some discrepancies between predicted and measured values, the general trend is reasonably well predicted. These predictive methods are particularly useful for assessing the impact of design changes on acoustics.

ACKNOWLEDGMENTS

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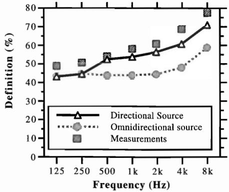
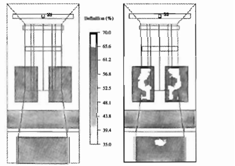


Figure 9 Effects of noise source.



(a) existing room (b) room with architectural modifications.

Figure 10 Predicted contours for Definition at 1 kHz.

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