

# Cyclostationarity for ship detection using passive sonar: progress towards a detection and identification framework

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## ABSTRACT

As the blades of a propeller pass through the water they produce characteristic amplitude modulated random noise signals which can be detected using sonar. It has recently been shown that the cyclostationary properties of this signal can be exploited to detect the presence of the propeller craft in significant extraneous noise. A detection technique based on the Cyclic Modulation Spectrum was shown to offer advantages over existing detection techniques in that no user interaction was required to design band pass filters, and superior frequency resolution was available to more accurately identify shaft and propeller pass frequencies. This technique has subsequently been developed to further exploit the cyclostationary properties of the signal by designing statistical thresholds which support automatic detection. This paper provides an overview of the progress of the cyclostationary detection work presented to date, and introduces a further development: exploiting cyclostationarity to determine the range, heading and speed of the surface ship. This concept is based on array processing using the cyclic autocorrelation function. The performance of this technique is demonstrated using simulation and the work is placed in the context of an overall detection and identification framework.

## INTRODUCTION

This paper presents an overview of the progress made in developing a detection and identification framework based on cyclostationary signal processing. The first section provides a summary of the detection methodology based on the Cyclic Modulation Spectrum (CMS) (Hanson et al, 2008), which form the basis for all subsequent developments. The second section briefly summarises the statistical thresholds developed to support automatic detection (Antoni and Hanson, 2009), and the third section presents the foundations for estimating the range, heading and speed of the ship based on cyclostationary detection-of-arrival estimation.

### 1. CYCLOSTATIONARITY FOR DETECTION OF PROPELLOR CRAFT

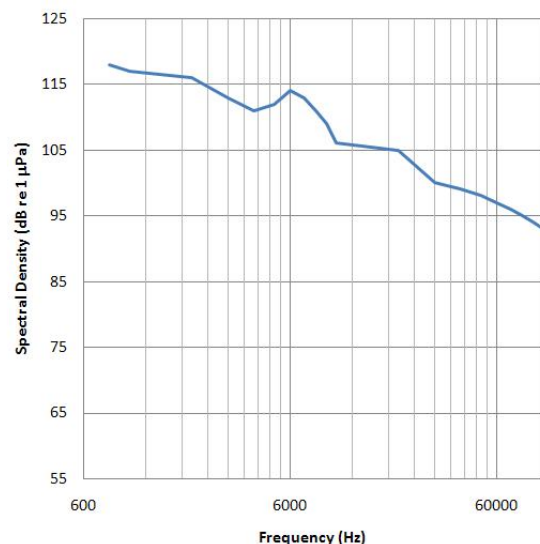
Of key interest to submariners is the ability to detect the presence of surface ships while remaining undetected themselves. To this end, passive detection techniques have been developed whereby the surface ship is detected, and in some cases classified, based on its noise emissions which are recorded by hydrophones on the submarine.

Detection is impeded when the signal from the ship is lost in the noise, which can occur when the marine environment is particularly noisy and/or when the ship is distant from the submarine. In order to maintain contact with a target vessel, the detection must rely on some form of signal processing to enhance the acoustic signature of the ship and attenuate the extraneous components in the sonar signal.

#### Propellor Noise Signal

The principal source of acoustic energy in the propeller signal is provided by cavitation (Sharma *et al*, 1990). Cavitation is a process whereby bubbles are drawn out of the water by pressure gradients on the blade surface and edges. These

bubbles are unstable, and it is their collapse that produces the noise. The spectral content of propeller cavitation noise is quite broadband, with significant energy out to at least 100kHz, as shown in **Figure 1**.



**Figure 1** Typical propellor noise spectrum (reproduced from data scaled from Fig. 5 in Sharma *et al*, 1990)

The degree of cavitation is related to the water pressure which varies with depth. Therefore, the cavitation noise will modulate as the propeller blade rotates through varying water depth. The propeller signal is thereby comprised of amplitude modulated cavitation components, with a modulation period akin to the propeller frequency. Expressed another way, the propeller signal can be seen to be made up of a broadband carrier component modulated by the periodic blade rotation. The challenge faced by passive detection techniques is to extract the periodic component.

## The Search for Hidden Periodicity

The quest for passive identification of propeller craft from sonar signals can be restated as the search for hidden periodicity in (presumably) uncorrelated noise. The periodicity arises from the rotation of the propeller blades, and the uncorrelated and often broadband noise is a feature of the marine background acoustic environment.

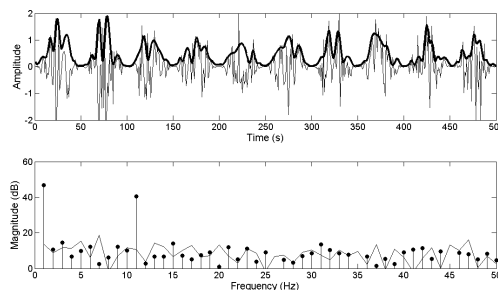
Signals combining random and periodic components are commonly encountered in e.g. mechanical applications. Machines with rotating components such as shafts, gearboxes and bearings are common, and a principle feature of condition monitoring of gearboxes and bearings, structural dynamics involving rotating machinery and vehicle dynamics, is the separation of these random and periodic components. The simplest technique for achieving this separation is time synchronous averaging. This involves taking the ensemble average of sections of the signal which are equal in length to the period of the harmonic component of interest. All other components are diminished by the averaging, leaving only the periodic component (see e.g. Peeters *et al.*, 2007). This technique demands that the period is both precisely known and constant, which may not be the case in blind mechanical applications such as passive detection. If the cycle could be readily identified, then order tracking could be employed to overcome any variation in the period of the harmonic component, but this is unlikely to be the case in such a noisy environment.

Another popular technique for the separation of random and periodic components has been self-adaptive noise cancellation (Antoni and Randall, 2004a) which in recent times has been refined in the form of the Discrete-Random Separator (DRS) (Antoni and Randall, 2004b). The DRS exploits the difference in correlation length between the random and periodic components in the signal to design an H1 style filter which extracts the periodic components in the frequency domain. In this way it is not susceptible to changes in the cycle of the periodic component, and does not rely on the precise identification of its period. Its utility was demonstrated through industrial applications involving modal analyses of a paper machine (Antoni, *et al.*, 2004) and a stadium cantilever stand (Hanson *et al.*, 2007).

Another technique for separating periodic components from broadband noise is liftering in the cepstrum domain. In the cepstrum of such a signal, the periodic components would manifest as a train of harmonics which can be removed by liftering. The signal can then be transformed back to the frequency or time domain and will comprise of only its random constituents. This technique has been applied to echo removal, double bounce removal from impact hammer response signals, and many machine condition monitoring applications (see e.g. Randall, 2000 and Gao and Randall, 1996). Indeed, it is already used in underwater acoustics to remove the first reflection components from sonar signals recorded in shallow water (Coates, 2001).

A different technique, developed empirically in the field of underwater acoustics, is known as DEMON processing (Detection of Modulation On Noise) which is employed by submariners to detect the presence of propeller craft. It uses the FFT of the envelope of band pass filtered sonar signals to emphasise the modulation in time of the pressure signal (see e.g. Coates 2001). This technique appears to be largely empirical, with few papers in the literature. Recently an extension to the technique was made by Li and Yang (2007) who employ higher order statistics to suppress Gaussian noise.

An example of the principles of DEMON processing is shown in Figure 3. Here is represented an amplitude modulated broadband signal and its corresponding spectrum, which is basically white in the frequency range of interest, i.e. the modulation does not manifest as identifiable harmonics. Also shown is the envelope of the band pass filtered signal, and its spectrum, in which the first harmonic of the modulation frequency is clearly evident. By DEMON processing therefore, the periodic modulation is transformed into a discrete frequency component which can be identified in the spectrum.



**Figure 2** periodically modulated broadband signal (top), its spectrum (solid line) and the spectrum of its envelope (dots) (bottom)

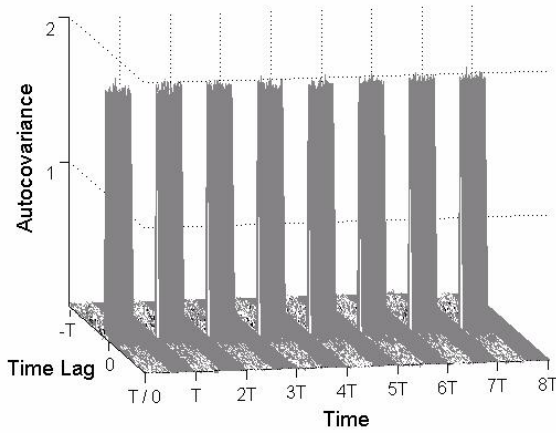
Expressed in another way, DEMON processing approaches the search for hidden periodicity phrased above, by attempting to transform the second order periodicity inherent in the propeller signal to first order periodicity through the signal envelope. By so doing, long established time-frequency signal processing techniques can be applied. Significant advantages exist however, in utilising the second order periodicity explicitly. Indeed, an entire toolbox is made available by recognising that the amplitude modulated propeller signal belongs to a special class of signals known as cyclostationary.

## Cyclostationarity

The term “cyclostationary” refers to a special class of non-stationary signals which are random in nature, but exhibit periodicity in their statistics. A first order cyclostationary signal (CS1) will exhibit periodicity in its first order statistics, i.e. its ensemble mean will be periodic; at the second order, its autocovariance. Consider a burst random signal; the first order statistics of the signal, i.e. the ensemble average over one on / off cycle, is zero and so not periodic. However the autocovariance (a second order statistic) of the signal, as represented in **Figure 3**, can be seen to exhibit periodicity in time  $t$ . Therefore, this signal may be described as second order cyclostationary (CS2).

Of particular importance to this work is the cavitation of propeller blades as they pass through the water. Like the burst random signal above, the cavitation from each blade pass may be considered as broadband and random, but they occur in a periodic fashion related to the shaft speed and are cyclostationary at the first and higher orders.

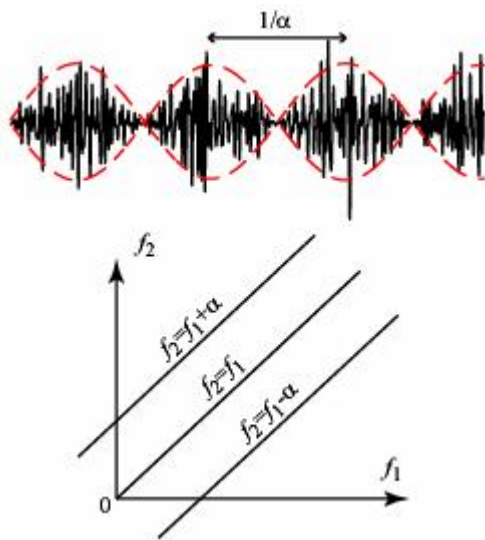
An important property of a cyclostationary signal is the cyclic period “ $T$ ”, which is the period of repetition observed in the statistics and is often described by its frequency domain analogue, the cyclic frequency “ $\alpha$ ”, where  $\alpha = \frac{1}{T}$ .



**Figure 3** Periodicity in the autocovariance of a second order cyclostationary signal

**Correlation in the Frequency Domain**

Another way to examine cyclostationarity is through the concept of correlation in the frequency domain, as explained by Antoni (2008). Antoni examines a stationary random carrier signal modulated by a harmonic function of period  $1/\alpha$ , as shown in **Figure 4**.



Source: (Antoni, 2008)

**Figure 4** Correlation in the frequency domain of a random carrier signal with harmonic modulation

Given that the signal is stationary random, we would expect that non-zero correlation exists only for the case where  $f_1 = f_2$ , i.e. where the corresponding frequency components in the two signals are aligned in the correlation. Indeed, as **Figure 4** reveals, the two signals exhibit correlation along the line  $f_1 = f_2$ . In addition however, it can be seen that correlation exists for either signal shifted by  $\alpha$  (nb: actually any integer multiple of  $\alpha$ ). Antoni expresses this relationship in reverse, explaining that it is the spectral components spaced apart by  $\alpha$  which are interfering in such a way to produce the periodic modulation in the time domain.

This correlation can be exploited to identify the propeller components in a noisy sonar signal. Rather than focusing on the temporal evolution of the modulation (cyclic) frequency,

i.e. time-frequency spectrum, further insight can be gained by examining the frequency-cyclic frequency spectrum.

This work makes use of the CMS, which is calculated from the DEMONgram (time-frequency spectrum) by taking the Fourier transform of the squared signal along the time axis:

$$P(\alpha, f) = \mathfrak{F}_{t \rightarrow \alpha} \{ X(t, f)^2 \} \quad (1)$$

where  $\mathfrak{F}_{t \rightarrow \alpha}$  means the Fourier transform from time  $t$  to frequency  $\alpha$  and  $X(t, f)$  is the short-time Fourier transform of signal  $x$  centered around time  $t$ .

This produces a two dimensional spectrum in terms of frequency and cyclic-frequency (frequency shift). The frequency domain correlation manifests as a non-zero spectrum at the modulation frequency, which may contain useful spectral information in its own right. Only the magnitude is examined here however, as the intent is limited to detection, rather than identification.

The cyclostationary technique employed in this investigation can be summarised in the simple block diagram shown in **Figure 5**.



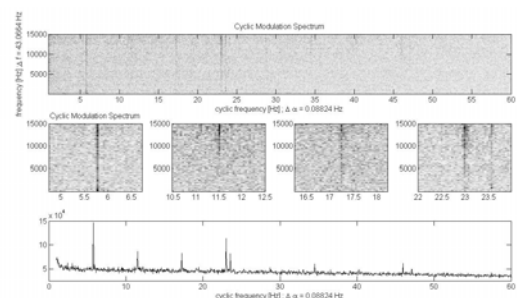
**Figure 5** Schematic of cyclostationary propeller detection process

It should be noted that the same uncertainty principle applies to the CMS as to the instantaneous power spectrum, i.e. the reciprocal of the time resolution  $\Delta t$  is the largest cyclic frequency  $\alpha_{max}$  that can be identified:

$$\alpha_{max} \leq 4\pi\Delta f \quad (2)$$

**Results**

The cyclostationary detection technique was applied to a sonar signal of a North Sea Coaster, recorded by Professor Rodney Coates off the coast of the United Kingdom. The results of this analysis are presented in **Figure 6**. The CMS clearly identifies the shaft speed and its harmonics, and the blade pass frequency.



**Figure 6** CMS of the North Sea Coaster signal (top), zoom on each harmonic (middle) and mean cyclic frequency spectrum (averaging along frequency axis) (bottom).

The fine (cyclic) frequency resolution afforded by the CMS makes it possible to discriminate paired harmonics of shaft speed and blade pass – perhaps associated with two propeller shafts operating at slightly different speeds (in this case approximately 8rpm). Indeed, this technique allows the modulation to be detected over a wide frequency region whilst maintaining the fine cyclic frequency resolution. Techniques such as DEMON processing however, require that the modulation be detected over a relatively narrow frequency range, thus requiring significant guess work in designing the filter pass-band.

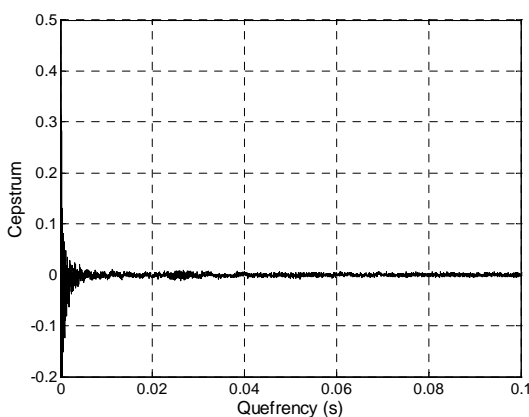
### The CMS and the Cepstrum

The cepstrum is sometimes defined as the inverse Fourier transform of the squared signal:-

$$c_x = \mathfrak{F}^{-1} \left\{ \log \left( |X^2| \right) \right\} \quad (3)$$

In this way, the CMS and the cepstrum (apart from the log operation) have been confused in previous presentations of this work. The difference between the CMS and the cepstrum, apart from the log operation, is the axis along which the second Fourier transform is applied. In the cepstrum, the inverse Fourier transform is applied along the frequency axis, therefore the cepstrum of a time vs frequency spectrum would produce a time-queffrequency spectrum, i.e. the time evolution of the cepstrum of the signal. The CMS by contrast produces a frequency vs frequency-shift spectrum, and in this way is able to discriminate components in the signal which exhibit correlation in the frequency domain. First order cyclostationary components, i.e. periodic signals, would be revealed in the cepstrum as a train of harmonics, but second order components such as the amplitude modulation in the propeller signal would not produce harmonics, just as they do not produce harmonics in an ordinary frequency spectrum. In this way, only the CMS can reveal the presence of the propeller signal.

This is shown by way of example in **Figure 7** which presents the cepstrum of the North Sea Coaster signal, calculated in accordance with Eq. 3.



**Figure 7** Cepstrum of the North Sea Coaster signal

The shaft and propeller components which are clearly evident in the CMS are entirely absent from the cepstrum.

## 2. STATISTICAL THRESHOLDS FOR AUTOMATIC DETECTION

An extension to the cyclostationary detection technique, and an advance toward autonomous detection, is the establishment of statistical thresholds for cyclostationarity that will

indicate the presence of a ship. Thresholds for detecting the presence of cyclostationarity in signals have been developed previously for applications in e.g. telecommunications (Zivanovic and Gardner, 1991) and machine condition monitoring (Raad *et al* 2008). Zivanovic and Gardner propose a DCS (Degree of Cyclostationarity) measure based on the distance between the nonstationary autocorrelation and the nearest stationary autocorrelation. For a stationary process, the DCS = 0, whereas for cyclostationary signals, DCS > 0. Along similar lines, Raad *et al* presented an Indicator of Cyclostationarity (ICS) for cyclostationarity of order  $n = 1-4$ , which is based on the  $n$ th order cumulants in the signal. In this case, at the second order the ICS represents the energy normalised autocovariance of the signal, which for  $\alpha > 0$  reduces to zero for the null hypothesis that the signal is stationary.

The approach outlined below, and explored more fully in Antoni and Hanson (2009) is based on the Cyclic Modulation Coherence, and provides an exact threshold of cyclostationarity as a function of all the computation parameters. It has the advantages that it closely mimics statistical tests based on the cyclic spectral coherence which are optimal in several instances, it is extremely fast to compute, and it applies whenever the cyclic frequency bandwidth to scan is not greater than about 4 times the frequency resolution  $\Delta f$ , which is a common situation in most sonar applications.

### Definition of the cyclic modulation coherence

The cyclic modulation coherence (CMC) is a power-normalised version of the CMS. The CMC intends to approximate the cyclic spectral coherence while offering a much faster way of computation by making a systematic use of the discrete Fourier transform (DFT) and its related FFT algorithm.

The CMC is defined as

$$CMC(\alpha, f) = \frac{P_x(\alpha, f)}{P_x(0, f)}, \quad (4)$$

where  $P_x(\alpha, f)$  was the CMS. The CMC therefore is the CMS normalised by the estimated PSD so as to eliminate all scaling effects. Therefore the capability of detecting the presence of cyclostationarity in some frequency band will not depend on the actual energy level in that band, but only whether energy fluctuates therein periodically or not. Note that the normalisation is also equivalent to computing the CMS of the whitened signal, which is a customary preprocessing step in most detection tests.

### The CMC in the presence of cyclostationarity

In order to understand the behaviour of the CMC in the presence of cyclostationarity - and in particular how it relates to the cyclic spectral coherence - let us investigate its expression in the simple case where signal  $x(n)$  exhibits cyclostationarity at a single and arbitrary cyclic frequency  $\alpha_0$ . Under some mild assumptions it can be shown that

$$CMC(\alpha, f) = D_l(R\alpha) + \frac{A_0(f)}{2S_x(f)} D_l(R\alpha - R\alpha_0) + O_p(I^{-1}) \quad (5)$$

where

- i) the Dirichlet kernel,  $D_I(R\alpha) = e^{-j\pi\alpha R(I-1)} \mathbf{K}^{-1} \sin(\pi\alpha RI) / \sin(\pi\alpha R)$  is the only function of  $\alpha$  and therefore controls the cyclic frequency resolution, i.e.  $\Delta\alpha \sim 1/IR \sim 1/L$  as returned by the width of its main lobe,
- ii) the ‘‘cyclostationary’’ signal-to-noise ratio (SNR)  $0 \leq A_0(f)/2S_x(f) \leq 1$  returns the strength of the cyclostationary component relatively to the signal power at frequency  $f$ ,
- iii) the residual estimation noise  $O_p(I^{-1})$  has magnitude that vanishes like  $1/I$  in probability.

Therefore, because  $D_I(R\alpha)$  rapidly tends to a train of discrete delta functions with period  $1/R$ ,

$$CMC(\alpha, f) \rightarrow \begin{cases} 1, & \alpha = 0 \\ \frac{A_0(f)}{2S_x(f)}, & \alpha_0 + p/R, \text{ for any interger } p \\ 0, & \text{elsewhere.} \end{cases} \quad (6)$$

This is fine enough to detect the presence cyclostationarity at  $\alpha = \alpha_0$ , but not fully satisfying since the CMC also returns non-zero values at all other cyclic frequencies  $\alpha = \alpha_0 + p/R$ , thus erroneously indicating other cyclostationary components where there are not. The reason stems from undersampling the STFT by factor  $R$  which entails frequency aliasing in  $\alpha$ . In order to gain more insight into this issue and see how to solve it, it must be realised that  $A_0(f)$  in Eq.6 is obtained as

$$\frac{A_0}{2} = \kappa_w(\alpha_0) S_x(\alpha_0, f), \quad (7)$$

where  $\kappa_w(\alpha) = R_w(\alpha_0)/R_w(0)$  with  $R_w(\alpha) = \sum_{n=0}^{N-1} w(n)^2 e^{-j2\pi\alpha n}$  and where  $S_x(f, \alpha_0)$  is the cyclic power spectrum defined as [6]

$$S_x(f, \alpha_0) = \lim_{N \rightarrow \infty} \lim_{I \rightarrow \infty} \frac{1}{I \cdot R_w(0)} X_N(t_i, f) X_N^*(t_i, f - \alpha_0) \quad (8)$$

with \* the complex conjugate symbol. In Eq.8,  $\kappa_w(\alpha_0)$  is seen to act as a low-pass filter in  $\alpha$  which gradually brings down  $A_0(f)$  to zero as  $\alpha_0$  increases. For instance, for a  $N$ -long Hanning window,  $\kappa_w(0) = 1$  and  $\kappa_w(|\alpha_0| \gg \alpha_{\max}) \approx 0$  with  $\alpha_{\max} \sim 4/N$ . The reason for this becomes clear if one construes the STFT  $X_N(t_i, f)$  as a narrow band-pass filtered signal in band  $[f - \Delta f/2, f + \Delta f/2]$  where  $\Delta f \sim 1/N$ . Hence, the narrower the band, the slower the variations of the energy flow  $|X_N(t_i, f)|^2$  through it, with cut-off frequency  $|\alpha| \leq \alpha_{\max} \sim 4\Delta f$ . The existence of such a cut-off frequency is actually a chance to reject the undesirable aliased

cyclic frequencies  $\alpha = \alpha_0 + p/R$ ,  $|p| > 1$  in Eq.6. This is achieved provided that  $1/R > \alpha_{\max}$ , i.e.  $R \leq N/4$  with a Hanning window, meaning that at least 75% overlap should be set when computing the STFT. The CMC is then non-zero at  $\alpha = 0$  and  $\alpha = \alpha_0$  only, thus detecting cyclostationarity at the correct location.

### Statistical test

The CMC was demonstrated to correctly detect the presence of cyclostationarity in a limited cyclic frequency range. In most sonar applications this will be fine enough, since the cyclic frequency of interest will usually be much smaller than the coarser allowable spectral resolution, i.e.  $|\alpha_0| < \Delta f$ , which is consistent with the previous requirement that  $|\alpha_0| < \alpha_{\max}$  with  $\alpha_{\max} \sim 4/N$ . Moreover, the CMC being a complex quantity in general, its squared-magnitude will be used for detection. The image formed by  $|CMC(\alpha, f)|^2$  as a function of  $\alpha$  and  $f$  will then provide a good and fast *visual test* to check for the presence of cyclostationarity in the signal. Once a frequency band  $[f_1, f_2]$  is identified where cyclostationarity is present, a better *statistical test* is then given by the integrated squared-magnitude CMC (ICMC), namely,

Reject the null hypothesis  $H_0$  ‘‘there is no presence of cyclostationarity at cyclic frequency  $\alpha_0$  (i.e.  $A_0(f) = 0$ )’’ at the  $p$  level of significance if :

$$\frac{1}{K} \sum_{k=k_1}^{k_2} |CMC(\alpha_0, f_k)|^2 > \lambda_{1-p}(\alpha_0), \quad (9)$$

where  $K = k_1 - k_2$  with  $k_1$  and  $k_2$  the DFT bins corresponding to  $f_1$  and  $f_2$  and  $\lambda_{1-p}(\alpha_0)$  a statistical threshold to be determined. By allowing the user to select a relevant frequency band where the cyclostationary SNR is high, the proposed statistical test will be all the more efficient. In addition, because of the integration over frequency  $f$ , the test will amount to comparing a function of  $\alpha$  only against the threshold  $\lambda_{1-p}(\alpha)$ . It now remains to find the level of that threshold in the general case, as a function of the number  $K$  of integrated frequency bins, the number  $I$  of signal blocks, the shift  $R$  between adjacent blocks, the length  $N$  and the type of the analysis window  $w(n)$ .

### Statistical threshold under the null hypothesis

The statistical threshold  $\lambda_{1-p}(\alpha)$  reflects that level the ICMC should not exceed with a risk probability  $p$  when the signal is assumed stationary at cyclic frequency  $\alpha$ . It may be found as follows. First, it is noticed that the CMC is asymptotically complex Gaussian with zero mean and variance

$$\sigma^2(\alpha) = \frac{1}{RI} \sum_{m=1-N}^{N-1} \left| \sum_{n=\max(0, m)}^{\min(N-1, N-1+m)} w(n)w(n-m) \right|^2 e^{-j2\pi\alpha m}, \quad |f| > 1/R, \quad (10)$$

The normalised squared-magnitude CMC is then distributed as  $\sigma^2(\alpha)/2 \cdot \chi_2^2$ , where  $\chi_2^2$  is a Chi2 variable with 2 degrees of freedom. Since the frequency bins of the DFT are

asymptotically independent under  $H_0$  (Brillinger, 2001), the sum of  $K$  Chi2 variables with 2 degrees of freedom then follows another Chi2 variable with  $2K$  degrees of freedom. Hence,

$$\frac{1}{K} \sum_{k=1}^{k=2} |CMC(\alpha, f_k)|^2 \sim \frac{\sigma^2(\alpha)}{2K} \chi_{2K}^2 \quad (11)$$

where symbol  $\overset{d}{\sim}$  means “distributed as”. Based on this result, the statistical threshold is finally found as  $\lambda_{1-p}(\alpha) = \chi_{1-p,2K}^2 \cdot \sigma^2(\alpha) / (2K)$  with  $\chi_{1-p,2K}^2$  the  $100(1-p)$ th percentile of the  $\chi_{2K}^2$  distribution.

**Practical recommendations**

This paragraph aims at summarising the main recommendations for an optimal use of the proposed statistical test:

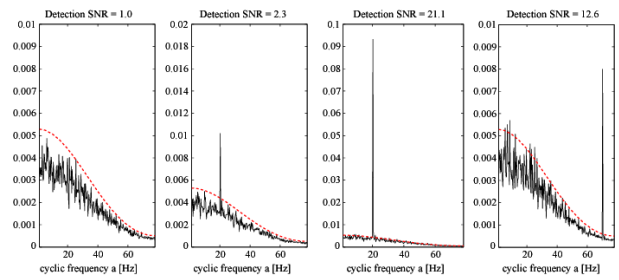
- Disregard the first cyclic frequency  $\alpha = 0$  which always returns  $CMC(0, f) = 1$  and may then absorb most of the dynamical range of the CMC.
- Set at least 75% overlap when computing the STFT with a Hanning window (the question as which optimal window to be used for minimising cyclic leakage is currently under investigation).
- If leakage is suspected, use a different fraction of overlap and check whether the suspicious peak remains at the same cyclic frequency or not.
- On the CMC displayed as an image in the  $(\alpha, f)$  plane, select a frequency band  $[f_1, f_2]$  where the cyclostationary SNR is maximised.
- Compute the ICMC in that frequency band and compare the result against the statistical threshold for a given  $p$  level of significance.
- Eventually compute the “detection” SNR  $CMC(\alpha_0, f) / \lambda_{1-p}(\alpha_0)$ , a useful indicator for appraising the significance of the detection.

As a final remark, it should be noted that the proposed statistical test does not strictly test for the presence of cyclostationarity, but rather for the absence of stationarity -- this is a common feature of all similar tests that have been proposed in the literature. Consequently, the ICMC is likely to exceed the statistical threshold whenever the signal exhibits nonstationarity, but not necessarily cyclostationarity. However, it has been proved in [6] that cyclostationarity only can produce significantly high magnitudes of the cyclic spectral coherence, and therefore of the CMC. In that sense the detection SNR  $CMC(\alpha_0, f) / \lambda_{1-p}(\alpha_0)$  may prove very useful.

**Results**

The application of the statistical thresholds described above is demonstrated here with an example. This simulation considers four signals, each of length 32768 samples with sampling frequency 10kHz; a) a stationary signal, b) a cyclostationary signal with  $\alpha_0 = 20$  Hz and SNR = -10dB, c) a cyclostationary signal with  $\alpha_0 = 20$  Hz and SNR = 0dB, and d) a cyclostationary signal with  $\alpha_0 = 70$  Hz and SNR = 0dB.

The integrated CMC of each of these signals is shown in Figure 8 where the detection performance in case (b) is particularly noteworthy considering the very low SNR in this case.

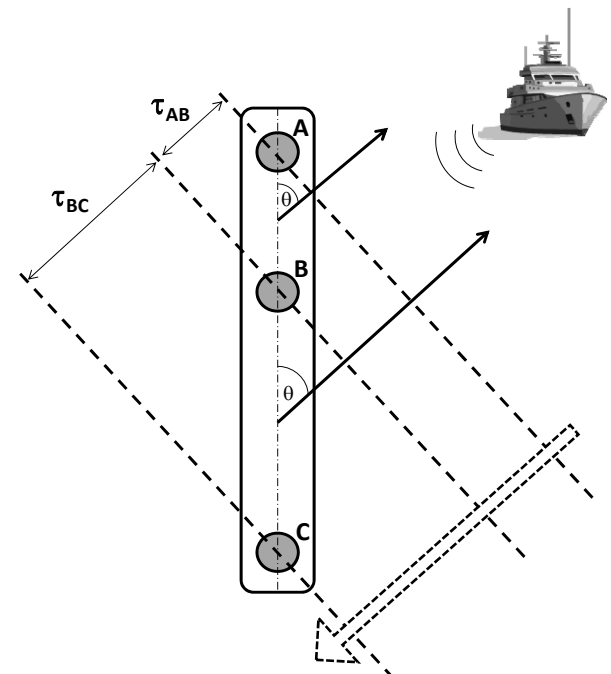


**Figure 8:** integrated squared-magnitude cyclic modulation coherence in the band [1000,3000]Hz in the four cases of Fig.3 together the statistical threshold (red dotted line) at the 0,5% level of significance.

**3. DIRECTION OF ARRIVAL ESTIMATION USING CYCLOSTATIONARITY**

Cyclostationary signal processing can also be applied to the problem of Direction-Of-Arrival (DOA) estimation, through which the bearing of a ship can be estimated using array processing. DOA estimation based on cyclostationarity has previously been applied in telecommunications in which signals are frequently inherently cyclostationary as modulations in frequency, amplitude and phase are employed to convey information. Digital signals can also be rendered cyclostationary through oversampling. The concept presented below approaches the problem from a fundamental perspective, but future work will involve adapting learnings from telecommunications for use in this sonar application.

A schematic overview of the DOA estimation problem is shown in Figure 9 in which the sound waves from the ship are incident on the sonar array as plane waves with a certain time delay between each hydrophone.

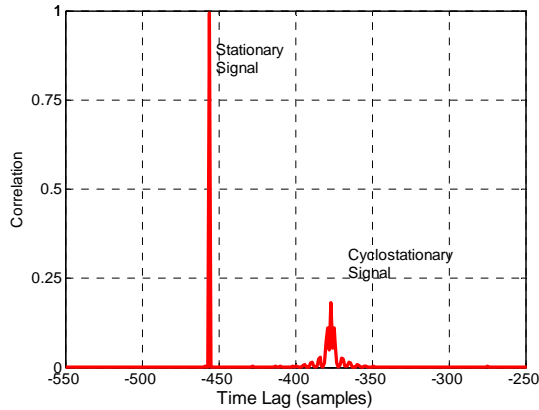


**Figure 9** A schematic representation of the DOA problem, wherein the ship is sufficiently far from the submarine for the sound waves to approximate plane waves

This time delay can be estimated using the cross-correlation function or from the phase of the cross spectrum between the signals received by each hydrophone.

Successful estimation of the time delay depends on clear definition of the coherent signal components from the ship at

each receiver sensor. Furthermore, the sensors will receive signals from each independent noise source in the vicinity, complicating estimation of the time delays associated with the signals from the ship. This problem is illustrated in **Figure 10**, which shows an example of a cross correlation between two simulated sensor signals – each receiving a signal from a stationary source and an independent and non co-located cyclostationary source.



**Figure 10** Cross correlation between two simulated signals

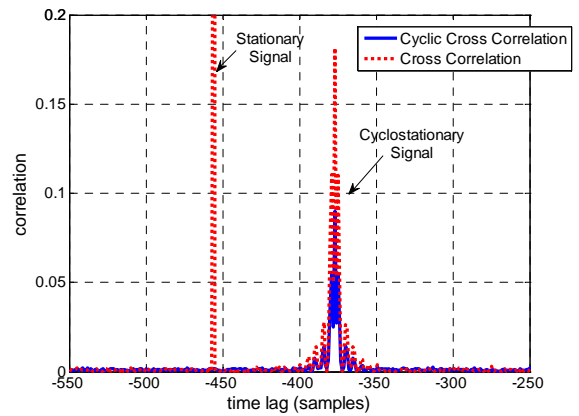
The cross correlation exhibits characteristic peaks at time lags corresponding to the delay time between the two sensors of both the stationary signal and the cyclostationary signal. In this particular case, the stationary signal produces the highest correlation, thereby potentially confounding an automated time delay estimation algorithm.

This problem can be overcome using cyclostationary signal processing if the cyclic frequency of the ship is known. Rather than utilising the cross correlation (or cross spectrum if operating in the frequency domain), the time delay estimation can make use of the cyclic cross correlation, i.e. the cross correlation computed at the frequency shift corresponding to the cyclic frequency of the ship’s propeller signal. The cyclic cross correlation is given by:-

$$R_{XY}[\tau; \alpha_i] = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} R_{XY}[n, \tau] e^{-j2\pi\alpha_i n \Delta} \tag{12}$$

where  $R_{XY}[n, \tau]$  is the cross-correlation function between signals  $X$  and  $Y$ ,  $\alpha_i$  is the  $i$ th multiple of the cyclic frequency and  $\tau$  is the time lag. The cyclic frequency of the ship propeller signal could be identified from the CMS during the initial detection operation.

The effectiveness of the cyclic cross correlation is shown in **Figure 11** which compares the cross correlation from **Figure 10** with the equivalent cyclic cross correlation.



**Figure 11** Cross correlation (red - dash) and cyclic cross correlation (blue – solid) between two simulated signals

The cyclic cross correlation exhibits a peak only at the time delay associated with the cyclostationary source, as the stationary source does not correlate with itself at the cyclic frequency under examination.

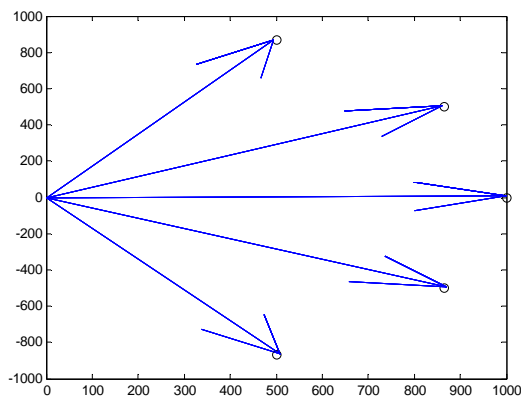
The cross correlation can be efficiently estimated as the inverse Fourier transform of the cross spectrum. It was previously shown that preweighting the cross spectrum by the signal coherence and extracting the phase component yields a clearer peak in the cross correlation and hence a superior time delay estimation (Gao *et al* 2006). This concept was extended in Hanson *et al* (2007) in which time delay estimation as employed for detecting the presence and location of leaks in underground water pipes. In (reference) it was shown that the phase cepstrum offered superior time delay estimation as the time delay manifested as a series of harmonics, rather than a single peak as in the cross correlation. Although beyond the scope of this preliminary investigation, cyclic analogies of both the enhanced cross correlation estimators and the phase cepstrum could be applied for cyclostationary DOA estimation.

Once the time delay between sensor pairs have been estimated, the bearing to the ship can be calculated from:-

$$\sin \theta = \frac{\tau c}{D} \tag{13}$$

where  $D$  is the distance between the sensors. Note that each paired combination of sensors will yield a bearing estimate which, if the sensor array is sufficiently large, could be used to determine the range to the ship by triangulation.

The effectiveness of the DOA estimation concept was demonstrated using a simulation of a ship at a nominal distance of 1000m from an array of three sensors. The ship was positioned at thirty degree increments with respect to the sensor array and the bearing to the ship estimated in each case. The results of this simulation are shown in **Figure 12**.



**Figure 12** results of the direction-of-arrival simulation; location of the ship (circles) and estimated bearing from submarine (arrows)

As these preliminary results reveal, the cyclic DOA estimation technique was successful in identifying the location of the ship in each case. Note that cyclostationarity is not able to resolve left-right ambiguity in the DOA estimate.

Once the bearing and range of the ship had been identified, the speed could be estimated by calculating the change in these quantities over time. Thus, the bearing, speed and range of the ship could all be identified through passive sonar techniques. These concepts remain to be further explored.

## DISCUSSION AND FUTURE WORK

This paper presented an overview on progress made to date in the development of a passive sonar technique for the detection of surface ships from submarines based on cyclostationarity. The fundamental detection process, based on the cyclic modulation spectrum, was outlined and demonstrated using measured signals. A statistical threshold based on the integrated cyclic modulation coherence was presented and applied to simulated signals. Finally, a concept for direction-of-arrival estimation was proposed and demonstrated using a simple simulation. It was explained how this concept could be expanded to also estimate the range, heading and speed of the ship.

The focus of future work will be to further develop the statistical thresholds so as to allow automatic detection of surface ships, and integration with the DOA estimation and CMS tools to provide further information on the ship. The ultimate aim is to develop a fully integrated tool which will also move some way to identifying the type of ship that has been detected.

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