System Identification with Mean Differential Cepstrum using Random Decrement

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ABSTRACT

This paper looks at the recently developed Mean Differential Cepstrum (MDC) method for calculating frequency response functions (FRF) from response signals. It has been shown to work in single-input and multiple-input scenarios, giving both magnitude and phase information. Its applications have been largely confined to transient signals, in accordance to the original definition of the MDC. The key motivation for this paper is to extend this blind system identification method to continuously excited systems, like in most machinery in practice. The use of the Random Decrement Technique (RDT) to pre-condition continuous signals, prior to applying the MDC method, is investigated. The idea is to utilise the impulse like derived signals for the identification process. Qualitative comparisons between identification outcomes using RDT derived signals and actual transient signals are presented for single-input systems.

INTRODUCTION

Operational Modal Analysis (OMA) is increasingly prevalent over experimental modal analysis (EMA) for assessment of structural dynamic properties. This trend can be attributed to the significant cost associated with generating input forcing signals and the difficulties with measuring input signals in the operating environment. The common OMA approaches like commercially available techniques: Polymax (Peeters *et al.*, 2004) and Frequency Domain Decomposition (FDD) (Brincker *et al.*, 2000), use the response cross-spectral matrix (CSM) in place of frequency response functions (FRFs) for analysis. The CSM does not give single-input single-output (SISO) information in a multiple-input multiple-output (MIMO) set-up. Consequently, SISO curve fitting techniques (Gao and Randall, 1996) used for extracting scaled modeshapes under non spectrally white excitations, cannot be employed.

The Mean Differential Cepstrum (MDC) methodology presented was developed for the identification of MIMO systems from response signals (Chia, 2007). It gives SISO information like the FRFs and is capable of handling "non-minimum phase" systems. The identified magnitude and phase information are used in place of FRFs for modal analysis. The development began from the scalar definition of the MDC (Antoni *et al.*, 2000) and led to the definition of propagative solution sequences for both multiple-input and single-input scenarios.

The method has been successfully applied in single-input systems using transient input signals (Chia *et al.*, 2007). This paper looks into extending the application to continuous signals using the Random Decrement Technique (RDT). Continuous signals generated from a simulated single-input system are preconditioned with the RDT before applying the MDC method. The identification outcome is compared to results from transient input signals for a qualitative assessment.

BACKGROUND

The Cepstrum and Differential Cepstrum are the origins of the MDC. Their definitions and applications are introduced together with a brief description of the RDT.

Cepstrum

The term Cepstrum was first coined in 1963 and was initially developed for echo detection (Bogert *et al.*, 1963). In the cepstral domain, the source and transmission path effects in a single input system can be separated when the input has a smooth spectrum (Randall, 1987). The homomorphic nature of Cepstrum, accords the input and system an additive relation as illustrated in **(1)**. The Cepstrum of the generic system equation, $Y(\omega) = H(\omega)X(\omega)$ gives:

$$\begin{aligned}
\mathbf{C_y}(\tau) \;=\; Y(\tau) &= \mathscr{F}^{-1}\left(ln\, Y(\omega)\right) \\
&= \mathscr{F}^{-1}\left(ln\, (H(\omega)X(\omega))\right) \\
&= \mathscr{F}^{-1}\left(ln\, H(\omega) + ln\, X(\omega)\right) \\
&= \mathscr{F}^{-1}\left(ln\, H(\omega)\right) + \mathscr{F}^{-1}\left(ln\, X(\omega)\right) \\
Y(\tau) &= H(\tau) \;+\; X(\tau)
\end{aligned} \tag{1}$$

where	X:	Input	t:	time
	Y:	Output	ω:	frequency
	H:	System	τ:	quefrency (units of time, s)
			$\mathscr{F}$:	Fourier transform

The Cepstrum can be defined as the inverse Fourier transform of the logarithmic spectrum. Its homomorphic nature is the consequent of the logarithmic operator. Examples of its application include separation of glottal excitation and vocal tract impulse response in speech analysis (Noll, 1964) (Oppenheim and Schafer, 1968), separation of excitation and structural responses in gear boxes (Randall, 1984) and blind system identification (Hanson *et al.*, 2007)

Differential Cepstrum

The Differential Cepstrum (DC) is defined as the inverse Fourier transform of the derivative of a logarithmic spectrum (Polydoros and Fam, 1981):

$$\mathbf{d_y}(\tau) = \mathscr{F}^{-1}\left(\frac{d}{d\omega}\, ln Y(\omega)\right) = \mathscr{F}^{-1}\left(\frac{Y'(\omega)}{Y(\omega)}\right) \tag{2}$$

The DC has the added advantage of not requiring phase unwrapping needed in the Cepstrum, but retains the homomorphic nature. Similar to the Cepstrum, it is utilised for echo removal (Polydoros *et al.*, 1979). Note that most mechanical system are "minimum phase" systems where phase information can be obtained using the inverse Hilbert transform (Papoulis, 1962)(Randall, 1987). The challenge comes when dealing with "non-minimum phase" systems.

Mean Differential Cepstrum

The Mean Differential Cepstrum (MDC) is defined as the inverse Fourier transform of the partial derivative of the logarithmic Spectral Correlation Density (SCD)(Antoni *et al.*, 2000) and can be shown to give:

$$\mathbf{d_{yy}}(\tau) = \mathscr{F}^{-1}\left(\mathbf{D_{yy}}(\omega)\right) \tag{3}$$

where
$$\mathbf{D_{yy}}(\omega) = \frac{\mathbf{E}\left(Y'(\omega)Y^*(\omega)\right)}{\mathbf{E}\left(Y(\omega)Y^*(\omega)\right)} \tag{4}$$

is the frequency domain MDC while $\mathbf{E}(...)$ is the expected value and superscript * denotes conjugate. It is developed for a series of stochastic/random transient signals, allowing for ensemble averaging to remove noise. Note that **(3)** is identical to **(2)** for a single realisation.

In subsequent mathematical expressions, the (ω) term will be dropped for visual simplification and frequency domain operation is assumed unless otherwise stated.

The MDC method developed led to two approaches of system identification: direct and indirect. In single-input systems, only the former is relevant. The following shows the development of the direct approach in the matrix form.

The following are assumed about the input excitation sources:

- mutually uncorrelated sources
- spectrally white input
- by convention have unitary power (scale indeterminacy) (i.e. all input sources are assumed to have a power of one. This assigns any gain to the transfer functions (FRFs) from each source to each response DOF)

The formulation of the propagative solution sequence, for the solution of the system matrix $\mathbf{H}$, starts with the matrix MDC of the output response vector $\tilde{\mathbf{Y}}$:

$$\mathbf{D_{yy}} = \mathbf{E}\left(\tilde{\mathbf{Y}}'\tilde{\mathbf{Y}}^H\right)\ \mathbf{E}\left(\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^H\right)^{-1} \tag{5}$$

Substituting $\tilde{\mathbf{Y}} = \mathbf{H}\,\tilde{\mathbf{X}}$,

$$\begin{aligned}
\mathbf{D_{yy}} &= \mathbf{E}\left(\left(\mathbf{H}'\tilde{\mathbf{X}} + \mathbf{H}\tilde{\mathbf{X}}'\right)\left(\tilde{\mathbf{X}}^H\mathbf{H}^H\right)\right) \\
&\qquad \mathbf{E}\left(\left(\mathbf{H}\tilde{\mathbf{X}}\right)\left(\tilde{\mathbf{X}}^H\mathbf{H}^H\right)\right)^{-1} \\
&= \left(\mathbf{H}'\underbrace{\mathbf{E}\left(\tilde{\mathbf{X}}\tilde{\mathbf{X}}^H\right)}_{\mathbf{S_{xx}}=\mathbf{I}}\mathbf{H}^H + \mathbf{H}\underbrace{\mathbf{E}\left(\tilde{\mathbf{X}}'\,\tilde{\mathbf{X}}^H\right)}_{jC\left[\ddots\right]}\mathbf{H}^H\right) \\
&\qquad \left(\mathbf{H}\underbrace{\mathbf{E}\left(\tilde{\mathbf{X}}\tilde{\mathbf{X}}^H\right)}_{\mathbf{S_{xx}}=\mathbf{I}}\mathbf{H}^H\right)^{-1}
\end{aligned}$$

taking into account the identity matrix term $\mathbf{S_{xx}} = \mathbf{I}$,

$$\mathbf{D_{yy}} = \left(\mathbf{H}'\mathbf{H}^H + \mathbf{H}\,jC\left[\ddots\right]\mathbf{H}^H\right)\left(\mathbf{H}\mathbf{H}^H\right)^{-1}$$

post multiplying with $(\mathbf{H}\mathbf{H}^H)$,

$$\begin{aligned}
\mathbf{D_{yy}}\left(\mathbf{H}\mathbf{H}^H\right) &= \left(\mathbf{H}'\mathbf{H}^H + \mathbf{H}\,jC\left[\ddots\right]\mathbf{H}^H\right) \\
\mathbf{D_{yy}}\,\mathbf{H} &= \mathbf{H}' + \ \mathbf{H}\,jC\left[\ddots\right]
\end{aligned}$$

$jC\left[\ddots\right]$ is a purely imaginary diagonal matrix and can be shown to correspond to time displacements (Chia *et al.*, 2005), which can be set to zero since there is no absolute time reference in response measurements.

$$\mathbf{H}' - \mathbf{D_{yy}}\,\mathbf{H} = 0 \tag{6}$$

Defining the derivative as a backward difference with respect to frequency ω, (6) can be manipulated to give:

$$\begin{aligned}
\mathbf{H}' &= \mathbf{D_{yy}}\,\mathbf{H} \\
\frac{\mathbf{H}(\omega_k) - \mathbf{H}(\omega_{k-1})}{d\omega} &= \mathbf{D_{yy}}(\omega_\mathbf{k})\,\mathbf{H}(\omega_k)
\end{aligned}$$

multiplying by $d\omega$ and isolating the $\mathbf{H}(\omega_k)$ term,

$$\left(\mathbf{I} - d\omega\,.\,\mathbf{D_{yy}}(\omega_\mathbf{k})\right)\mathbf{H}(\omega_k) = \mathbf{H}(\omega_{k-1})$$

$$(\mathbf{H_{DirOrn}}) \quad \mathbf{H}(\omega_k) = \left(\mathbf{I} - d\omega\,.\,\mathbf{D_{yy}}(\omega_\mathbf{k})\right)^{-1}\ \mathbf{H}(\omega_{k-1}) \tag{7}$$

(7) is a matrix equation which solves for the system matrix $\mathbf{H}$ in a propagative manner. The (kth) frequency bin value is calculated based on the previous, ($k-1$). $\mathbf{H}$ and the matrix multiplicative factor has the same square dimension. The direct approach based on the above original formulation can be adapted by approximating the two terms in the multiplicative factor to a Taylor series:

$$(\mathbf{H_{DirTay}}) \quad \mathbf{H}(\omega_k) = \left(e^{-d\omega.\mathbf{D_{yy}}(\omega_k)}\right)^{-1}\ \mathbf{H}(\omega_{k-1}) \tag{8}$$

(7) and **(8)** are the basis of the propagative direct identification process, using the original $(\mathbf{H_{DirOrn}})$ and the Taylor series adapted $(\mathbf{H_{DirTay}})$ formulations respectively. The identified systems are in complex numbers where magnitude and phase information are directly obtained. For single-input applications, the scalar response signal Y is used in place of the vector $\tilde{\mathbf{Y}}$.

Random Decrement Technique

The Random Decrement technique was introduced (Cole, 1968) for the purpose of extracting impulse response like signals from continuous signals.

The deterministic portion of a signal would become more prevalent with each Random Decrement averaging, producing a distilled, impulse response like signal. The selection of time history sections for ensemble averaging depends on the trigger condition. They are chosen to reflect the response history of interest and are typically:

a) zero crossing with a steep positive slope
b) zero crossing with a steep negative slope
c) crossing of a constant amplitude level

All these cases resemble initial responses from an impulse. The first two can be used in conjunction by using the same slope magnitude as the trigger and inverting the signal with the initial negative slope before ensemble averaging. The last trigger condition based on a constant amplitude level is illustrated in **figure 1**.

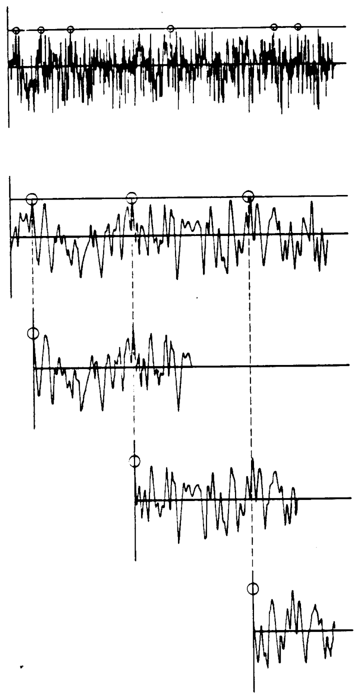

Figure 1: Random Decrement using constant amplitude trigger

SIMULATION SET-UP

Figure 2 shows the simulated 5 degrees of freedom (DOF) system. Its mass and stiffness matrices are based on the system properties depicted. Responses from m_3, the driving point measurement, were used for identification.

The two input excitation cases at X were:
(i) 10 minute continuous random excitation and
(ii) 100 realisations of 4s burst random excitation (BR4s)

A positive constant amplitude level crossing was the RDT trigger condition used in case (i). A slight modification was to begin data capture at the positive zero-crossing immediately prior to the trigger level. This level is at 50% maximum response signal amplitude. A total of 550 sets of 16s long signals were extracted.

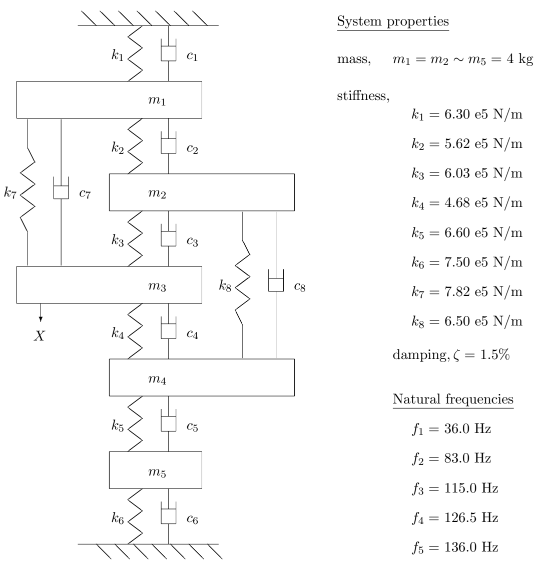

Figure 2: Simulation set-up (5 DOF system)

Based on 50 averages, 11 realisations of impulse-like signals were derived and applied to the identification process. Similarly for case (ii), each of the 100 response realisations is 16s long.

RESULTS

Response Time History

Figure 3 shows a RDT derived response from case (i). A rapid decay from the start of the signal for approximately 0.5s is observed. The response remains within a constant amplitude band after this, with no further indication of amplitude decay. The signal within this band resembles a scaled version of the system's response to continuous excitation.

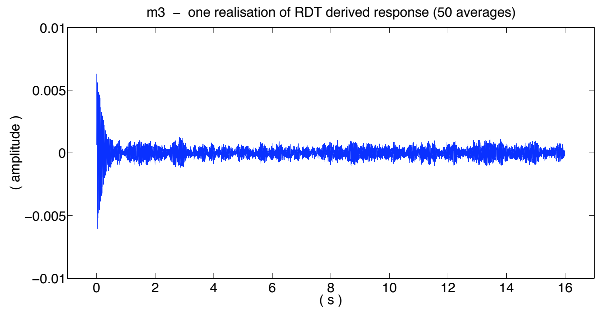

Figure 3: RDT derived response (50 averages)

In **figure 4**, the first four seconds reflects the system's response to random excitation from the 4s burst random transient input. A rapid decay characteristic similar to that in **figure 3** is observed after the 4th second. The amplitude reduces to almost zero after 2s of decay.

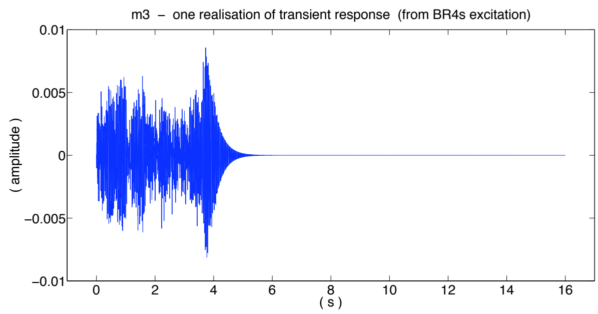

Figure 4: Response from BR4s transient excitation

Both response time histories are plotted on the same scale for comparison. On inspection, the initial amplitude decay on both signals are similar, which is not unexpected given they reflect the same system. The main observed differences are the decay to zero amplitude and the high amplitude response (initial 4s) observed only in **figure 4**.

Identified Systems

Figures 5 and **6** are the direct MDC identification outcome using responses to continuous excitation and RDT, and using transient BR4s excitation, respectively. Both magnitude and phase information are presented in the same figure.

In each figure, results from the original $(\mathbf{H_{DirOrn}})$ and the Taylor series approximated $\left(\mathbf{H_{DirTay}}\right)$ formulations are superposed together with the actual FRF of the system $(\mathbf{H_{ref}})$. The identified systems are intentionally displaced to facilitate comparison. The absolute gain factor of the systems, relative to the input, is irrelevant for response only identification processes.

The following legend applies for both figures.

$\left(\mathbf{H_{DirOrn}} \text{ ——}, \ \mathbf{H_{ref}} \text{ ——}, \ \mathbf{H_{DirTay}} \text{ ——}\right)$

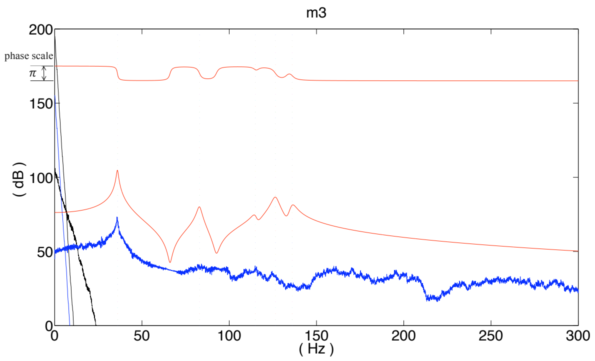

Figure 5: Identification from RDT derived responses

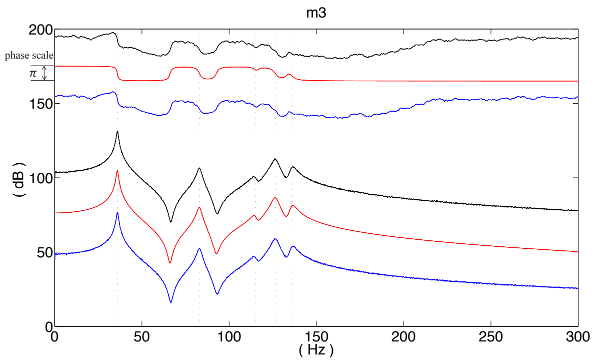

Figure 6: Identification from BR4s induced transient responses

The quality of the identification is reflected by how closely the shape of the identified magnitude and phase resembles those of the actual FRF. The identified system using the RDT derived responses does not resemble the actual FRF as closely as those derived from the use of transient responses.

In **figure 5**, only $(\mathbf{H_{DirTay}})$ produced a magnitude with some semblance to the actual FRF. The more stable solution outcome of $(\mathbf{H_{DirTay}})$ relative to $(\mathbf{H_{DirOrn}})$ was similarly observed in earlier work (Chia *et al.*, 2007).

Figure 6 clearly shows that the MDC method works well with transient excitations. The identified magnitudes are very similar to the actual FRF. The identified phases show clear and distinct transition at each pole and zero despite the "wavy" appearance. The minimum phase system characteristic, where phase decreases and increases by π at each pole and zero respectively, is apparent.

DISCUSSION

Further work is required to improve the identification outcome from distilling continuous signal using RDT and applying it to the MDC identification method. In this first encounter, it seems that amplitude decay to zero plays a key role in achieving quality identification outcome.

Considerations for refining the RDT process include the trigger condition, percentage of overlap between signals and optimum signal length to be used for identification process.

A higher constant level trigger condition, for example, would lead to a set of stronger signals being extracted . The smaller number of signals extracted, from a given continuous signal length, will potentially benefit from a better signal-to-noise ratio.

The percentage overlap between adjacent signals for the current extracted 550 sets of 16s long signals from a 10 minute data is >90%. For the same data length, only a maximum of 37 sets of non-overlapping 16s long signals can be extracted. A balance between having excessive redundant information in the former and sub-optimal use of information in the latter is needed.

The chosen signal length of 16s used in the simulation is arbitrary. The system's decay characteristics should have an influence on the optimum length to be used. From **figures 3** and **4**, the suspicion is that very little useful information can be gained beyond 2s of amplitude decay. A decreased signal length for the RDT process in this case would likely benefit from greater number of extracted signals with lower percentage overlap.

In the next step forward, changes to trigger conditions and signal length, and keeping the signal overlap in view will be explored. Applying a known exponential window to force amplitude decay on response signals is also potentially beneficial. The artificially decayed signal may lead to better identification quality. The effects of the exponential window can be removed in post-processing.

The key motivation for using RDT is the prospect of using the MDC identification method on continuous signals. The ultimate goal is to apply the the identification method on MIMO systems under continuous excitation.

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