Review of adaptive tuned vibration neutralisers

Carl Howard

School of Mechanical Engineering, The University of Adelaide, Adelaide, South Australia, Australia

ABSTRACT

Tuned vibration neutralisers and absorbers provide an effective method of attenuating tonal vibration within a structure. If the frequency of the tonal vibration alters, then an adaptive tuned vibration neutraliser or absorber can be utilised to track and adapt to changes in the frequency of the source vibration. This paper provides an overview of some of the issues associated with the development of adaptive tuned vibration neutralisers.

INTRODUCTION

Tuned vibration neutralisers (TVNs) and tuned vibration absorbers (TVAs) are devices that are attached to a (primary) structure to reduce its vibration. The idea was patented by Frahm in 1911 and has been used in automotive, marine and aerospace applications. They are a rigid mass attached to a primary structure through an elastic spring. The spring component has been implemented using several mechanisms such as: a cantilever beam, a curved beam, air-spring (bellows), shape-memory alloy beams, and so on. These devices have also been implemented to reduce torsional vibration, and in these applications, the spring element is realised using a torsional stiffness element.

Although the terms Tuned Vibration Absorber / Damper / Neutraliser are often used interchangeably, they can be differentiated by the mechanism that they operate to reduce the vibration of a primary structure (Bonello *et al.* 2005). If the device is installed to reduce a structural resonance in a primary structure, the device is tuned to a frequency slightly lower than the structural resonance frequency and is constructed with an appropriate amount of damping, it is called a tuned vibration *damper* or *absorber*. If the device is installed to reduce the vibration in a primary structure due to forced excitation at a particular frequency, the device typically has low damping to provide the greatest vibration attenuation, the resonance frequency of the device is tuned to the forcing frequency and is referred to as a tuned vibration *neutraliser* (Brennan 1997). The focus of this paper is the latter configuration for tuned vibration neutralisers.

EQUATIONS OF MOTION

Consider the system shown in Figure 1 of a tuned vibration neutraliser attached to a primary structure. The equations of motion for the system, which are available from most vibration textbooks, are given by

$$F_1 = -m_1\omega^2 x_1 + k_1 x_1 + j\omega x_1 c_1 + k_2(x_1 - x_2) + j\omega c_2 (x_1 - x_2) \quad (1)$$

$$0 = -m_2\omega^2 x_2 + k_2(x_2 - x_1) + j\omega c_2(x_2 - x_1) \quad (2)$$

where *m*, *k*, and *c* are the mass, stiffness and viscous damping coefficients, respectively, ω is the frequency is radians/sec, and F_1 is tonal force. By rearranging (2) to find an expression for x_2 and substituting it into (1), results in the expression

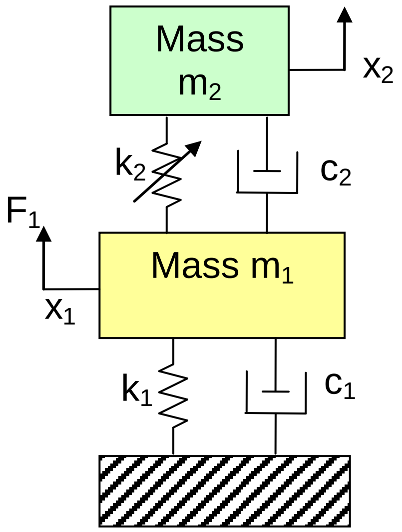

Figure 1. Model of an adaptive tuned vibration neutraliser attached to a vibrating primary structure.

$$\frac{x_1}{F_1} = \left[-m_2\omega^2 + k_2 + j\omega c_2\right] / \left[\begin{array}{l} -k_2 m_2\omega^2 - j\omega^3 c_2 m_2 + m_1\omega^4 m_2 \\ -m_1\omega^2 k_2 - m_1\omega^3 j c_2 - k_1 m_2\omega^2 \\ +k_1 k_2 + k_1 j\omega c_2 - j\omega^3 c_1 m_2 + \\ j\omega c_1 k_2 - \omega^2 c_1 c_2 \end{array}\right] \quad (3)$$

Hence, the primary structure is stationary (x_1=0) when the numerator in (3) equals zero. The numerator in (3) is recognisable as the form of the homogeneous differential equation of motion for a single-degree-of-freedom mass-spring-damper system. In other words, when the tuned vibration neutraliser is resonant (for the case of the single-degree-of-freedom system), the vibration of the primary structure is minimised. The *damped* resonance frequency of the TVN is given by

$$\omega_d = \omega_n \sqrt{1-\xi^2} \quad (4)$$

where $\omega_n^2 = k_2 / m_2$ is the *natural* frequency, and the damping factor is defined as $\xi = c_2 / \left(2\sqrt{k_2 m_2}\right)$.

Before discussing methods for adaptively altering the tuning the neutraliser, it is worthwhile to examine the vibration response of the system in the frequency range about its resonance frequency.

The vibration attenuation A of the primary system due to the attachment of the TVN is calculated as the ratio of the displacement after attachment divided by the displacement without the TVN as

$$A = x_1[\text{with TVN}] \,/\, x_1[\text{without TVN}] \tag{5}$$

The equation of motion of the primary system without a TVN is given by

$$F_1 = -m_1\omega^2 x_1 + k_1 x_1 + j\omega x_1 c_1 \tag{6}$$

Eq. (6) can be rearranged for the displacement x_1 normalised by the driving force F_1 as

$$\frac{x_1[\text{without TVN}]}{F_1} = \frac{1}{-m_1\omega^2 + k_1 + j\omega c_1} \tag{7}$$

Assuming that the driving force on the primary structure remains the same after the attachment of the TVN, the vibration attenuation from Eq. (5) is calculated as Eq. (3) divided by Eq. (7) and is given by

$$A = \frac{\left[-m_2\omega^2 + k_2 + j\omega c_2\right]\left[-m_1\omega^2 + k_1 + j\omega c_1\right]}{\left[\begin{array}{l} + m_1 m_2 \omega^4 - k_2 m_2 \omega^2 - k_2 m_1 \omega^2 - k_1 m_2 \omega^2 \\ - \omega^2 c_1 c_2 + k_1 k_2 - j\omega^3 c_2 m_1 - j\omega^3 c_1 m_2 \\ - j\omega^3 c_2 m_2 + j\omega k_2 c_1 - j\omega k_1 c_2 \end{array}\right]} \tag{8}$$

As an example, a system with the parameters listed in Table 1 will be examined. The acceleration responses normalised by the driving force F_1 of the primary structure x_1 and the TVN x_2 are shown in Figure 2. The primary structure has a resonance frequency of about 16Hz, so in a typical application, the lowest operating forcing frequency would be greater than 23Hz. It can be seen that the acceleration response above 30Hz is relatively flat, compared to the response below 30Hz. For this example, if a TVN were attached to the system with the parameters listed in Table 1, it can be seen that there is a significant decrease in the vibration of the primary structure at 50.3Hz. This corresponds to the resonance frequency of the TVN, when the TVN is attached to a *rigid base*. It is important to realise that the maximum response of the TVN does not occur when the vibration of the primary structure is minimised. Rather, the maximum vibration response of the TVN occurs at 53Hz. Hence an algorithm to neutralise the vibration of the base structure that utilises only the vibration amplitude of the TVN will not be effective.

Figure 3 shows the phase of the transfer function of the vibration of the TVN divided by vibration of the base x_2 / x_1.

Table 1: Parameters for example system.

Description	Variable	Value
Mass of primary	m_1	100 kg
Stiffness	k_1	10^6 N/m
Viscous Damping	c_1	5 sN/m
Mass of TVN	m_2	10 kg
Stiffness	k_2	10^6 N/m
Viscous Damping	c_2	5 sN/m

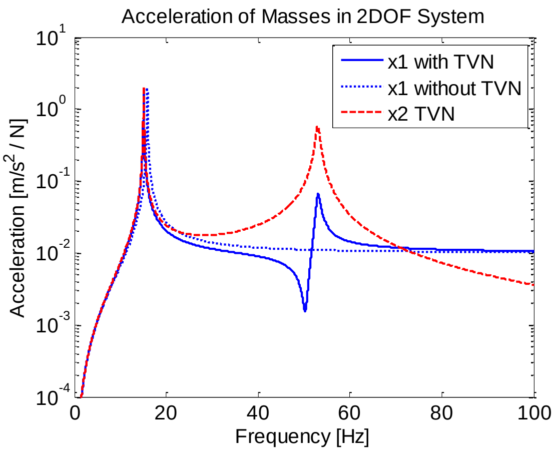

Figure 2: Normalised acceleration of the system with and without a TVN installed.

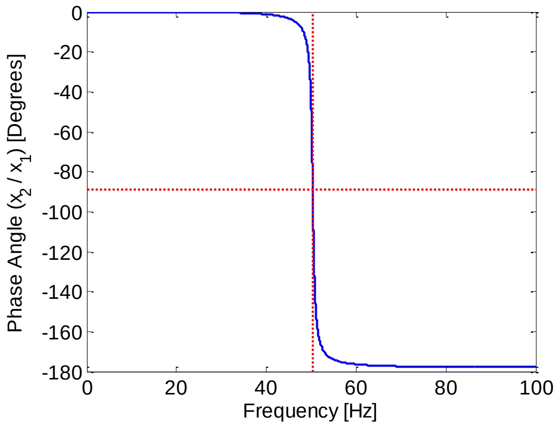

Figure 3: Phase response of the system.

At 50.3Hz there is a sharp change in the phase angle as it passes through 90 degrees, as indicated by the dotted lines, and hence there are some significant consequences. If a TVN is to be attached to a vibrating primary structure, it will only be effective if the driving frequency remains constant, which can be achieved with an electric motor. For other systems where the driving frequency can vary, the effectiveness of the TVN is reduced significantly for small changes in the driving frequency.

Figure 4 shows the ratio of the displacement of the base structure (x_1) with and without a TVN attached, calculated used Eq. (8) for three values of viscous damping in the TVN. It can be seen that the greatest vibration attenuation (smallest ratio of displacements) when there is minimal damping in the TVN, as shown by the thin solid line where x=50.3, y=0.014. The figure also indicates that for slight mistuning of the TVN from the driving frequency, the vibration attenuation is reduced. Unfortunately, increasing the damping of the TVN (c_2) does not widen the bandwidth of the TVN, and hence adding damping does not improve performance robustness to mistuning (von Flowtow et al., 1994). The only non-adaptive way to improve the robustness to mistuning is to increase the mass of the TVN such that even if the TVN is mistuned, it will still provide adequate vibration attenuation. However this is not an attractive alternative and hence the use of TVNs is not widespread. Hence, it would be beneficial to have a TVN where the resonance frequency of the device could be altered in response to changes in the driving frequency.

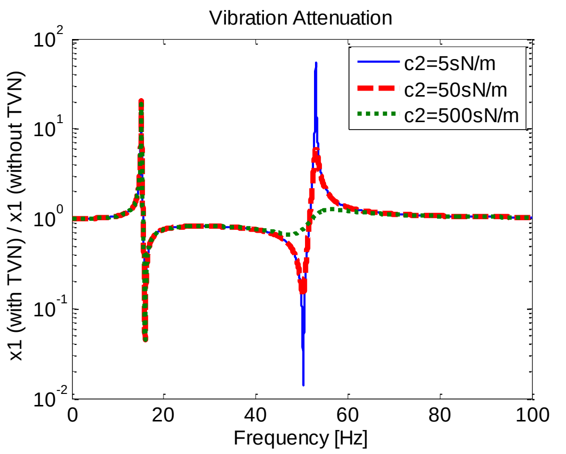

Figure 4: Ratio of displacements of the base structure x_1 with and without a TVN, from Eq. (8).

The following section describes several designs of adaptive vibration neutralisers and absorbers.

DESIGNS OF ADAPTIVE TUNABLE NEUTRALISERS AND ABSORBERS

The basic feature of an adaptive tunable vibration absorber or neutraliser is its capability to alter its resonance frequency. If the device has low damping then its resonance frequency is

$$\omega = \sqrt{k / m} \qquad (9)$$

where k is the stiffness and m is the mass of the vibrating part of the device. The resonance frequency of the device can be tuned by altering either the stiffness or the mass of the device. Interestingly, the author was unable to find any examples where the mass of the device was adjusted to change its resonance frequency, and hence provides an opportunity for future research. There are several designs in the literature that permit alteration of the stiffness of the device and are described below.

Beam Type Absorbers

Perhaps the most common implementation of the tunable vibration neutraliser is a mass on the end of a beam as shown in Figure 5. The cantilever beam and attached mass vibrate transversely. For this configuration, the position of the mass on the cantilever beam can be adjusted. The double-ended arrow indicates the direction of vibration of the base structure.

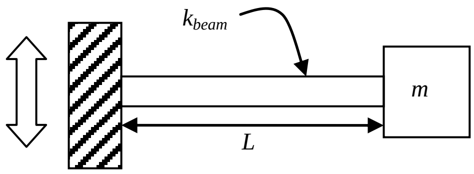

Figure 5: Cantilever beam type resonator.

The effective stiffness of the cantilever beam is

$$k = 3EI / L^3 \qquad (10)$$

where E is the Young's modulus, I is the second moment of area, L is the distance between base and the mass. Hence based on Eq. (10), the stiffness of the device can be altered by changing E, I, or L.

A design that permits a change in the second moment of area (I) of the beam is shown in Figure 6, where two leaf-springs are separated by an actuator which alters the distance between the two beams, and hence alters the effective second moment of area of the composite beam system (Walsh and Lamancusa, 1992).

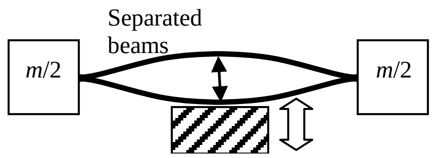

Figure 6: Resonator with separated beams.

The third parameter that can be altered is the Young's modulus of the beam. This implementation can be realised by using a shape-memory-alloy (SMA), which has the property that the stiffness of the material changes with temperature (Rustighi et al. 2003).

Curved Beams

Virgin and Davis (2003) described the use of buckled beams as a vibration isolator and Bonello *et al.* (2005) implemented this method in an adaptive vibration absorber, as shown in Figure 7. Bonello *et al.* used piezoceramic patches that were bonded to the curved beams so that radius of curvature of the beams could be altered by applying a voltage to the ceramic material.

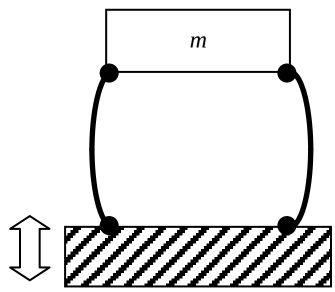

Figure 7: Resonator with curved beams.

Pneumatic Springs

Figure 8 shows an illustration of a resonator where the spring element is a pneumatic piston or an airbag. This type of device has been experimentally demonstrated by Brennan (1997).

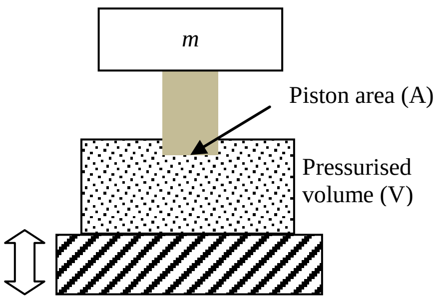

Figure 8: Illustration of a pneumatic spring isolator.

The effective stiffness k of a pneumatic spring is

$$k = \gamma P A^2 / V \qquad (11)$$

where P is the pressure in the device, A is the cross sectional area of the piston, V is the air volume within the device, and γ is the ratio of specific heat (=1.4 for air). The force exerted by the mass under gravity is balanced by the pressure acting over the face of the piston so that

$$mg = PA \quad (12)$$

The resonance frequency of the system is then

$$\omega = \sqrt{\gamma\, mgA/Vm} = \sqrt{\gamma\, gA/V} \quad (13)$$

which is independent of the mass *m* in the device.

This section has described several designs of devices that permit the alteration of their effective stiffness, so that their resonance frequency can be altered.

The following section contains a discussion of tuning algorithms that have been used with adaptive tuned vibration neutralisers.

TUNING ALGORITHMS FOR ADAPTIVE CONTROL

Cost Functions

A cost function is a metric that is used to evaluate the relative performance of the system. For example, the acceleration amplitude of the primary structure $|\ddot{x}_1|$ could be used as the cost function that should be minimised by a control system. The use of vibration amplitude as a cost function to be minimised requires the control system to continuously search to determine if the value of the cost function is minimised. For example, Figure 9 shows the same normalised acceleration response of the primary structure ($\ddot{x}_1 / F$) as shown in Figure 2, only over the frequency range from 40 to 60Hz.

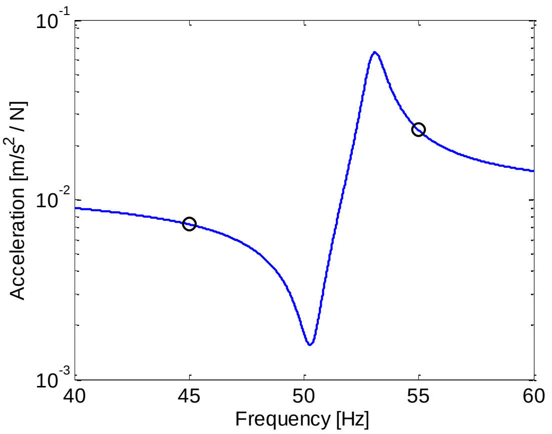

Figure 9: Normalised acceleration response of the primary structure ($\ddot{x}_1 / F$).

Consider that the driving frequency remains fixed at 45Hz, as indicated by the circle. Figure 9 shows that the configuration of the adaptive tuned vibration absorber is currently tuned to 50Hz, where the vibration is minimised. If the control system caused the stiffness of the TVN to increase, and hence the resonance frequency of the TVN would also increase, the curve would shift to the right and the amplitude of acceleration would increase. Presumably the control algorithm would recognise that the increase in vibration was undesired and should instead decrease the stiffness and hence resonance frequency of the TVN, so that the curve shifts to the left, until the trough in the response occurs at 45Hz. However once the system is tuned correctly, there is no guarantee that the vibration response is minimised. The control system would have to make small increases and decreases to the stiffness of the TVN so that the acceleration magnitude increased and decreased, and only then could the algorithm determine that indeed the acceleration response had been minimised. This description of the behaviour of the control system is similar to that which occurs in active vibration (and noise) control systems utilising gradient descent adaptive algorithms such as the Least-Mean-Squares algorithm.

Now consider that the excitation force was at 55Hz, as indicated by the circle in Figure 9. In this case, the control algorithm would attempt to reduce the acceleration of the primary structure by *decreasing* the stiffness and resonance frequency of the TVN so that the curve shifts to the left, which of course is incorrect. In order to properly tune the adaptive TVN to the excitation frequency, it would be necessary to *increase* the stiffness and resonance frequency of the TVN so that the curve shifted to the right. This example highlights the problem of a cost function and control system that is not able to tune to the *global* minimum of the cost function. Hence the use of vibration amplitude as a cost function is problematic.

To overcome this limitation, some researchers have used a control algorithm that comprises ‘rough’ and ‘fine’ tuning algorithms. The ‘rough’ tuning algorithm uses a ‘look-up’ table to adjust the stiffness of the system appropriately, and then switches to a ‘fine’ tuning algorithm that attempts to minimise the vibration of the primary structure. For this type of algorithm, a measurement is required that is related to the stiffness of the TVN. For example, when the TVN used is a mass on the end of a cantilever beam, the distance between the mass and the base of the cantilever is related to the stiffness of the TVN. A linear potentiometer could be used to measure the position of the mass.

The previous discussions highlight that in order to minimise the vibration of the primary structure an appropriate cost function is required. It is desirable to select a cost function that has a unique global maximum (or minimum). Two potential candidates are the use of the magnitude or phase angle of the transfer function between the vibration of the mass on the TVN and the primary structure. The phase response of the TVN shown in Figure 3 has a unique value of phase (-90 degrees) when the TVN is optimally tuned. However, the sharp change in the phase angle is problematic for a control system. A typically control system is implemented on a system that has a cost function that varies smoothly and slowly due to changes in operating conditions. In this case of implementing an adaptive TVN, the response of the system, which can be measured by either the vibration amplitude of the primary mass shown in Figure 2 or the phase angle shown in Figure 3, will change abruptly due to small changes the driving frequency.

A common method used to calculate phase angle of the TVN in ‘real time’ is the multiplication of the acceleration signals of the primary structure and the acceleration signal from the mass of the TVN, and then calculating a moving time average. For the case where the acceleration of the primary structure and the mass of the vibration absorber are given by

$$\ddot{x}_1 = \ddot{X}_1 \cos(\omega t) \quad (14)$$

$$\ddot{x}_2 = \ddot{X}_2 \cos(\omega t - \theta) \quad (15)$$

then the time average product of these two *tonal* signals is given by (Brennan et.al. 1996)

$$\overline{\ddot{x}_1\, \ddot{x}_2} = \frac{1}{T}\int_0^T \ddot{x}_1 \ddot{x}_2 dt = \frac{\ddot{X}_1 \ddot{X}_2}{2} \cos\theta \quad (16)$$

This calculation results in a signal that is ‘DC’ meaning that it is offset from zero and does not have sinusoidal components. It can be seen from Eq. (16) that the calculation results in a signal that is proportional to the cosine of the phase angle. When the TVN is optimally tuned, the phase angle will be θ=90 degrees, so cos(90)=0, and hence the time average product will equal zero. This method has been used by several researchers (for example, see Brennan *et al.* 1996, Long *et al.* 1995, Johnson *et al.* 2005).

As described previously, it is desirable to have a cost function that varies smoothly and slowly with changes in the operating parameters. For real systems, the vibration response of the primary structure $|\ddot{x}_1|$ and TVN $|\ddot{x}_2|$ is often unsteady and hence the phase angle will vary. Therefore it is important to select an appropriate averaging time T in Eq. (16) that is sufficiently long to remove short transients, but not excessively long such that the time taken to calculate the cost-function lags the actual phase angle of the TVN system.

Another practical consideration for calculating the phase angle is that vibration signals from real systems have noise and might have harmonic content. For these applications it is necessary to filter the vibration signals to remove the components that are not coherent with the tonal excitation frequency, which could be achieved using a tracking filter.

Adaptive Algorithms

As described in the previous sections, the response of the system changes rapidly due to changes in the stiffness of the TVN about the resonance frequency of the device. The control problem is therefore a non-linear optimisation problem and researchers have developed several control algorithms to tune TVNs.

Ryan *et al.* (1994) used a proportional gain feedback control system based on the acceleration amplitude of the base structure. In their system, the feedback error sensor was a rectified and low-pass filtered accelerometer signal and was used in an analog feedback circuit to minimise the vibration of the base structure. Their paper describes theoretical and experimental tests that demonstrate that the proposed control system was effective at reducing the vibration of the base structure.

Cronje *et al.* (2005) also used a feedback control algorithm using a displacement and velocity signals. Their tunable vibration isolator was unusual in that it used a wax actuator that was capable of exerting a force up to 500N and its properties were altered by a hot-air gun.

DiDomenico (1994) examined the use of a neural-network control system to reduce the vibration of a base structure. Control systems based on neural-network and genetic algorithms are good for optimising systems that behave non-linearly.

A third method that has been used involves a heuristic approach, called a ‘fuzzy’ controller, as proposed by Lai and Wang (1996). Their controller uses estimates of the total system energy that is to be minimised and provided some theoretical results. They also commented that in general the stability of fuzzy control systems cannot be guaranteed, which is unfortunate.

Long *et al.* (1995) and Brennan *et al.* (1996) used a combination of two control algorithms: a rough algorithm based on a lookup table of the required stiffness for a given excitation frequency, and then a fine tuning algorithm is used based on a steepest descent method. The algorithm to update the stiffness of the absorber is given by (Brennan *et al.*, 1996)

$$k_{\text{new}} = k_{\text{old}} + \mu \overline{\ddot{x}_1 \, \ddot{x}_2} \tag{17}$$

where μ is the convergence coefficient, and the expression $\overline{\ddot{x}_1 \, \ddot{x}_2}$ is calculated using Eq. (16) that provides a measure of the phase angle between the vibrating mass of the system and the base structure. When the device is tuned correctly, the phase angle is 90 degrees and the expression $\overline{\ddot{x}_1 \, \ddot{x}_2}$ is close to zero, hence the value of the new stiffness is the same as the old value of stiffness. Readers that are familiar with the Least-Mean-Squares (LMS) algorithm, may note that Eq. (17) is similar to the update equation for the LMS algorithm.

Nagata *et al.* (1999) used a simple control algorithm that measured the vibration of the base structure, altered the resonance frequency of the device, re-measured the vibration amplitude; if the vibration amplitude of the second measurement was smaller than the first, then the device would continue to alter the stiffness in the same sense otherwise the algorithm would change the stiffness of the device in the opposite sense.

A desirable feature of an adaptive algorithm for these devices is that it should not un-necessarily ‘hunt’ for the minimum value of the cost-function, which would prematurely wear the actuator mechanisms in the adaptive device. This can be achieved by adjusting the parameters in the algorithm such that when the cost-function is acceptably low, the actuators are not operated.

SUMMARY

This paper has provided a brief overview of some of the issues associated with the implementation of adaptive tunable vibration neutralisers. One of the inherent difficulties associated with tuning the device is the sharp change in the phase-response for small changes in the resonance frequency of the device. Several designs of adaptive tunable vibration neutralisers were presented that utilise a change in the stiffness of the device to provide the capability of varying its resonance frequency. Several adaptive control algorithms were described that can be used to tune the device to the driving frequency.

REFERENCES

Bonello, P., Brennan, M.J. and Elliott, S.J. 2005, *Vibration control using an adaptive tuned vibration absorber with a variable curvature stiffness element*, Smart Materials and Structures, 14, p1055-1065.

Brennan, M.J., Elliot, S.J., Long, T., 1996, *Automatic control of multiple tuned vibration neutralisers*, in Proceedings of Internoise 96, Liverpool, UK, p1597-1602.

Brennan, M.J., 1997, *Vibration control using a tunable vibration neutralizer*, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, **211** (2), p91-108.

Cronje, J.M., Heyns, P.S., Loveday, P.W., 2005, *Development of a variable stiffness and damping tunable vibration isolator*, Journal of Vibration and Control, **11**, p381-396.

DiDomenico, E., 1994, *Passive Vibration Tuning with Neural Networks*, Proceedings of Smart Structures and Materials, SPIE, p152-162.

Frahm, H. 1911, *Device for damping vibrations of bodies*, US Patent 989,958.

Johnson, M., Diggs, E.C., 2005, *Development of an analog controller for tuning an adaptive-passive control device*, Proceedings of 2005 ASME International Mechanical

Engineering Congress and Exposition, November 5-11, Orlando, Florida, USA, IMECE2005-82879.

Lai, J.S., Wang, K.W., 1996, *Parametric control of structural vibrations via adaptable stiffness dynamic absorbers*, Transactions of the ASME, Journal of Vibrations and Acoustics, 118, p41-47.

Long, T., Elliot, S.J., Brennan, M.J., 1995, *Semi-active control using tunable vibration absorbers*, in Proceedings of Internoise 95, Newport Beach, California, USA, p709-712.

Nagaya, K., Kurusu, A., Ikai, S., and Shitani, Y. (1999), *Vibration control of a structure by using a tunable absorber and an optimal vibration absorber under auto-tuning control*, Journal of Sound and Vibration, **228** (4), p773-792.

Rustighi E., Brennan, M.J. and Mace, B.R., 2003, *Design of an adaptive vibration absorber using shape memory alloy*, ISVR Technical Memorandum No. 920.

Ryan, M.W., Franchek, M.A., Bernhard, R., 1994, *Adaptive-passive vibration control of single frequency excitations applied to noise control*, in Proceedings of Noise-Con 94, Fort Lauderdale, Florida, USA, p461-466.

Virgin, L.N. and Davis, R.B., 2003, *Vibration isolation using buckled struts*, Journal of Sound and Vibration, **260** (5), p965-973.

Von Flowtow, A.H., Beard, A., Bailey, D., 1994, *Adaptive tuned vibration absorbers: tuning laws, tracking agility, sizing and physical implementations*, in Proceedings of Noise-Con 94, Fort Lauderdale, Florida, USA, p437-454.

Walsh, P.L. and Lamancusa, J. S. 1992, *A variable stiffness vibration absorber for the minimization of transient vibrations*, Journal of Sound Vibration, 158 (2), p195-211.