Tuned vibration absorbers for control of noise radiated by a panel

Swee S. Kuik, Carl Q. Howard, Colin H. Hansen & Anthony C. Zander

School of Mechanical Engineering, The University of Adelaide, Adelaide, S.A. 5005, Australia

ABSTRACT

A single passive tuned vibration absorber (TVA) that is specifically designed to achieve optimal performance at a particular frequency may not be effective in handling minor changes in excitation frequency. One way of taking this into account is to use multiple TVAs tuned to slightly different frequencies. This paper is concerned with the low- to mid-frequency vibrational behaviour and radiated noise of a panel excited by a point force and controlled using attached multiple vibration absorbers tuned to different frequencies. Finite element analysis is not commonly used to predict the effect of TVAs on the sound radiation by the panel. In this paper, a theoretical model and finite element models (including FEA with and without fluid-structure interaction) are presented for the calculation of the radiated sound power of a panel with multiple TVAs attached. A simply supported panel with two TVAs attached was used as an example to compute the theoretical predictions with the finite element analysis results.

1. INTRODUCTION

Tuned vibration absorbers (TVAs) have been used extensively for the control of tonal sound radiation from vibrating structures (Jolly & Sun 1996, Fuller *et al* 1997, Fuller & Cambou 1998, Marcotte *et al* 1999, Brennan & Dayou 2000, Wright 2003, Grissom 2003, Esteve 2004 and Howard 2008). The main reason for using TVAs rather than traditional damping treatment (Mead 1990, Gentry *et al* 1997 and Fuller *et al* 2004) is that they are light-weight and can be installed easily in engineering applications such as transportation and industrial equipment.

In this paper, an impedance approach is employed to describe the effect of TVAs attached to a simply supported panel and the results are compared with those obtained from finite element analysis.

The first part of this paper describes theoretical models to predict the dynamic response of and sound radiation from a simply supported panel with multiple attached tuned vibration absorbers. In the second part of the paper, results obtained using finite element models are compared with those obtained from theoretical models.

2. THEORETICAL BACKGROUND

The theoretical analysis presented in this paper is based on the work by Wright (2003), which is extended here to include the effect of panel damping. An impedance approach is used to describe the effect of TVAs attached to a simply supported panel as illustrated in Figure 1, where, for convenience only a representative single absorber is shown.

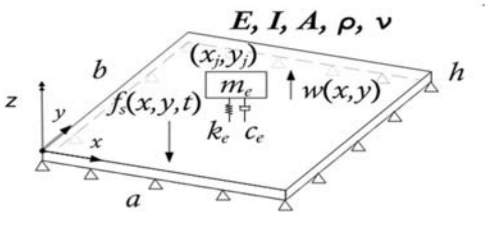


Figure 1. A schematic model of a simply supported panel with an attached TVA.

In the following section, analytical expressions for the dynamic response of a panel with attached TVAs will be introduced.

2.1 Multiple tuned vibration absorbers attached

For a thin homogenous panel excited by external forces, $f_s(x,y,t)$ and lying in the *x*-*y* plane, the two-dimensional bending wave equation in rectangular Cartesian coordinates can be written as (Soedel 1993)

$$EI\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^2 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho h\frac{\partial^2 w}{\partial t^4}=\sum_s f_s(x_s,y_s,t) \tag{1}$$

where E is Young's modulus, I is the second moment of area of the panel per unit width, $I = h^3/12\,(1-\upsilon^2)$, υ is Poisson's ratio, ρ is the density of the panel, and h is the thickness of the panel.

The quantity $w(x,y,t)$ is the transverse displacement of the panel surface at location *x*, *y* and time *t* and is given in modal terms by (Soedel 1993)

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\,\varphi_{mn}(x,y)\,e^{i\omega t} \tag{2}$$

where W_{mn} is the modal amplitude, φ_{mn} is the mode shape at location (*x*, *y*), $e^{i\omega t}$ represents the time dependence, and *m* and *n* are modal integers. The mode shape of a simply supported panel is given by (Soedel 1993)

$$\varphi_{mn}(x,y)=\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \tag{3}$$

where *a* and *b* are the length and width of the panel.

The modal amplitudes for a panel excited at frequency, ω can be expressed as (Wright 2003)

$$W_{mn} = \frac{4}{\rho h a b (\omega_{mn}^2 (1+i\eta) - \omega^2)} \sum_s F_s (x_s, y_s) \varphi_{mn} (x_s, y_s) \quad (4)$$

where η is the panel loss factor.

Eq. (2) can be written in matrix form as

$$w(x,y,t) = \mathbf{W}_{\mathbf{mn}}^{\mathrm{T}} \boldsymbol{\varphi}_{\mathbf{mn}} e^{i\omega t} \quad (5)$$

where the superscript T denotes the matrix transpose, and the mode shapes, $\boldsymbol{\varphi}_{\mathbf{mn}}$ can be expressed as a column vector and are arranged in order of increasing resonance frequencies, ω_{mn}, as

$$\boldsymbol{\varphi}_{\mathbf{mn}} = \begin{bmatrix} \varphi_{11} \\ \varphi_{12} \\ \vdots \\ \varphi_{mn} \end{bmatrix} \quad (6)$$

Finally, an expression for the modal amplitudes of a panel excited by point forces can be written in matrix form as (Wright 2003)

$$\mathbf{W}_{\mathbf{mn}} = \mathbf{H}_{\mathbf{mn}} \boldsymbol{\varphi}_{\mathbf{mn,s}} \mathbf{F}_{\mathbf{s}} \quad (7)$$

where $\mathbf{F}_{\mathbf{s}}$ is a vector of complex force amplitudes and $\mathbf{H}_{\mathbf{mn}}$ is a (m x n) diagonal matrix of containing the modal mobilities, which can be written as (Wright 2003)

$$\mathbf{H}_{\mathbf{mn}} = \begin{bmatrix} H_{11} & 0 & 0 & 0 \\ 0 & H_{12} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & H_{mn} \end{bmatrix} \quad (8)$$

Each element of $\mathbf{H}_{\mathbf{mn}}$ with included modal loss factor can be defined as

$$H_{mn} = \frac{4}{\rho h a b (\omega_{mn}^2 (1+i\eta) - \omega^2)} \quad (9)$$

which is an extension of the expression given by Wright (2003) with the loss factor excluded.

The resonance frequencies of the panel can be expressed as (Soedel 1993):

$$\omega_{mn} = \sqrt{\frac{EI}{\rho h}} \left(\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right) \quad (10)$$

The mode shape function is evaluated for each mode at the location corresponding to the s^{th} point force (i.e. s =1, 2, 3…n). Hence, it can be expressed as (Wright 2003)

$$\boldsymbol{\varphi}_{\mathbf{mn,s}} = \begin{bmatrix} \varphi_{11,1} & \varphi_{11,2} & \cdots & \varphi_{11,s} \\ \varphi_{12,1} & \varphi_{12,2} & \cdots & \varphi_{12,s} \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_{mn,1} & \varphi_{mn,2} & \cdots & \varphi_{mn,s} \end{bmatrix} \quad (11)$$

As shown by Wright (2003), the external forces, $\mathbf{F}_{\mathbf{s}}$ comprise the sum of the TVA reaction forces and disturbance forces. Hence, Eq. (7) can be simplified to give

$$\mathbf{W}_{\mathbf{mn}} = \mathbf{H}_{\mathbf{mn}} \boldsymbol{\varphi}_{\mathbf{mn,j}} \mathbf{F}_{\mathbf{TVAj}} + \mathbf{H}_{\mathbf{mn}} \boldsymbol{\varphi}_{\mathbf{mn,k}} \mathbf{F}_{\mathbf{d_k}} \quad (12)$$

where the disturbance forces can be written as a column vector as

$$\mathbf{F}_{\mathbf{d_k}} = \begin{bmatrix} F_{d_1} \\ F_{d_2} \\ \vdots \\ F_{d_k} \end{bmatrix} \quad (13)$$

and, the TVA reaction forces can be written as a column vector as

$$\mathbf{F}_{\mathbf{TVAj}} = \begin{bmatrix} F_{TVA_1} \\ F_{TVA_2} \\ \vdots \\ F_{TVA_j} \end{bmatrix} \quad (14)$$

Here, the complex amplitude of the j[th] TVA reaction forces can be expressed as (Wright 2003)

$$\mathbf{F}_{\mathbf{TVA\,j}} = i\,\omega\, \mathbf{Z}_{\mathbf{TVA\,j,j}} \boldsymbol{\varphi}_{\mathbf{mn,j}}^{\mathrm{T}} \mathbf{W}_{\mathbf{mn}} \quad (15)$$

where the superscript T denotes the matrix transpose. The TVA input impedance matrix can be written as

$$\mathbf{Z}_{\mathbf{TVA\,j,j}} = \begin{bmatrix} Z_{TVA_{1,1}} & 0 & 0 & 0 \\ 0 & Z_{TVA_{2,2}} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Z_{TVA_{j,j}} \end{bmatrix} \quad (16)$$

where each element in the matrix can be expressed as (Wright 2003)

$$Z_{TVA\,j,j} = m_{TVA\,j} \frac{\mathrm{i}\omega\omega_{TVA_j}^2 + 2\xi_{TVA_j}\omega^2 \omega_{TVA_j}}{(\omega_{TVA\,j} - \omega^2) - 2\mathrm{i}\xi_{TVA_j} \omega\omega_{TVA_j}} \quad (17)$$

where m_{TVA}, ξ_{TVA} and ω_{TVA} are the mass, critical damping ratio and resonance frequency of the tuned vibration absorber respectively. Also, $\boldsymbol{\varphi}_{\mathbf{mn,j}}$ can be expressed in matrix form as

$$\boldsymbol{\varphi}_{\mathbf{mn,j}} = \begin{bmatrix} \varphi_{11,1} & \varphi_{11,2} & \cdots & \varphi_{11,j} \\ \varphi_{12,1} & \varphi_{12,2} & \cdots & \varphi_{12,j} \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_{mn,1} & \varphi_{mn,2} & \cdots & \varphi_{mn,j} \end{bmatrix} \quad (18)$$

The matrix $\boldsymbol{\varphi}_{\mathbf{mn,k}}$ is defined in a similar way to Eq. (18) in which $\mathbf{j}$ is replaced by $\mathbf{k}$. The mode shape function is evaluated for each mode at the location corresponding to the s^{th} external force (which includes forces generated by the TVAs).

An expression for the modal amplitudes of the panel with TVAs attached can be derived from Eq. (12) and is given by Wright (2003) as

$$\mathbf{W}_{\mathbf{mn}} = \left[\mathbf{I} - \mathrm{i}\,\omega\, \mathbf{H}_{\mathbf{mn}} \boldsymbol{\varphi}_{\mathbf{mn,j}} \mathbf{Z}_{\mathbf{TVA_{j,j}}} \boldsymbol{\varphi}_{\mathbf{mn,j}}^{\mathrm{T}} \right]^{-1} \mathbf{H}_{\mathbf{mn}} \boldsymbol{\varphi}_{\mathbf{mn,k}} \mathbf{F}_{\mathbf{d_k}} \quad (19)$$

Finally, Eq. (19) can be substituted into Eq. (5) to estimate the dynamic response $w(x,y,t)$ of the panel. The mean square velocity of a panel can be expressed as follows (Wallace 1972):

$$\left\langle |\mathbf{v}|^2 \right\rangle = \frac{1}{8}\omega^2[\mathbf{w}]^{\mathrm{H}}[\mathbf{w}] \tag{20}$$

where the superscript H denotes the Hermitian transpose and **w** is defined in terms of elements defined in Eq. (5) as

$$\mathbf{w} = \begin{bmatrix} w(x_1, y_1) \\ w(x_2, y_2) \\ \vdots \\ w(x_g, y_g) \end{bmatrix} e^{i\omega t} \tag{21}$$

where the square of the surface velocity can be obtained by spatially averaging the squared velocity a number of g points spaced equally over the panel and using Eq. (20) to estimate mean square velocity of a panel.

2.2 Sound radiation from a vibrating panel

A schematic model of sound radiation from a simply supported panel mounted in an infinite baffle and subject to a point force excitation is shown in figure 2. The panel has a uniform thickness, h, width, a and length, b. The sound pressure field radiated by a vibrating surface surrounded by an infinite baffle can be calculated as (Wallace 1972)

$$p(r,\theta,\phi) = \int_S \frac{i\omega\rho_a \dot{w}(x,y) e^{i\omega t} e^{ikR}}{2\pi R} dS \tag{22}$$

where $w(x,y)$ is the component of the complex velocity normal to the surface, ρ_a is the density of the acoustic medium, ω is frequency in rad/s, r is the distance from the observation point to the coordinate origin.

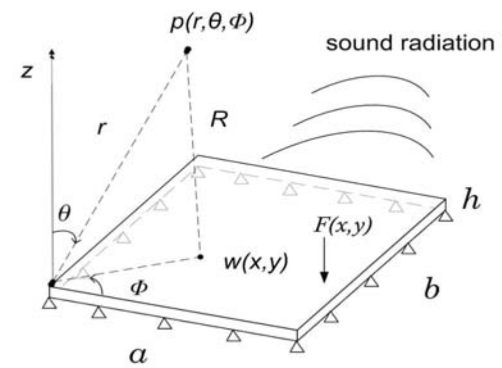


Figure 2. Schematic model of sound radiation of a simply supported panel with a point force excitation.

Panel vibration modes are orthogonal in terms of their vibration response. However, they are not orthogonal when describing their contributions to the radiated sound field. This simply means that the total radiated sound power cannot be evaluated by adding together all of the contributions from each mode. Basically, the integral can be estimated as the sum of the fields of a distribution of elemental sources on the radiating surface (each having a complex volume velocity) (Hansen & Snyder 1997).

The radiated sound intensity in the far field can be written as (Wright 2003)

$$I = |p(r,\theta,\phi)|^2 / (2\rho_o c) \tag{23}$$

Thus, the total radiated sound power can be written as (Wright 2003)

$$W = \int_0^{\pi} \int_0^{2\pi} I r^2 \sin\theta \, d\theta \, d\phi \tag{24}$$

3. FINITE ELEMENT ANALYSIS

Modelling a complete structure with attached tuned vibration absorbers can be done using the ANSYS finite element analysis (FEA) software package. A finite element model without and with fluid-structure interaction is as depicted in Figure 3 and Figure 4 respectively. A significant issue with conducting finite element analyses with fluid-structure interaction is the long computation time arising from non-symmetric matrix equation. This section presents a comparison between finite element analysis with and without fluid-structure interaction. The results from the analyses compare reasonably well with the theoretical predictions from Section 2.

A finite element analysis without fluid-structure interaction means that the finite element model comprises only structural elements. Finite element model as shown in Figure 3 only has shell (SHELL63), visco-elastic spring-damper (COMBIN14), and lumped mass (MASS21) elements. A harmonic response analysis was conducted using ANSYS to calculate the normal structural velocity distribution due to a point excitation force. The normal fluid velocities at the structure surface are assumed to be equal to the normal structural velocities. The ANSYS results of the structural velocity of the panel (without fluid-structure interaction) were exported and a MATLAB script was used to calculate the total radiated sound power.

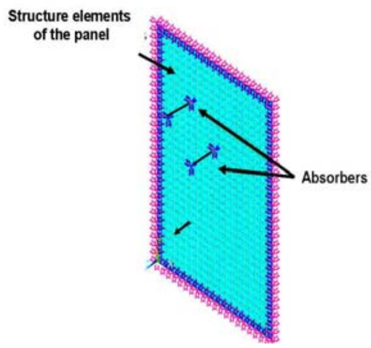


Figure 3. An FE model of the plate-mounted TVA system without fluid-structure interaction.

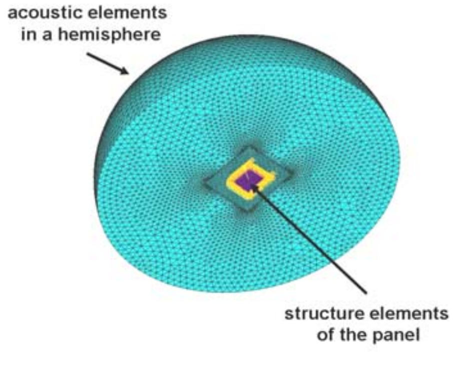


Figure 4. An FE model of the plate-TVA mounted system with fluid-structure interaction.

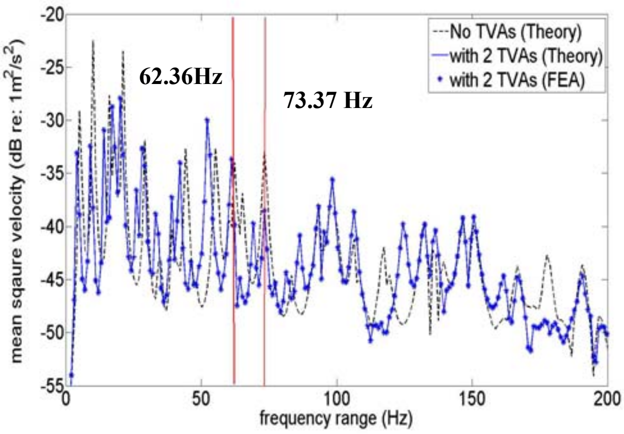
A finite element analysis with fluid-structure interaction has both acoustic (FLUID30 and FLUID130) and structure elements. Figure 4 shows the finite element model under consideration here that has a simply supported panel with TVAs attached, and is surrounded in a semi-infinite acoustic element hemisphere. The ANSYS software is used to calculate the radiated acoustic pressure from the panel. These pressure results were then exported and a MATLAB script was used to process the results to calculate the total radiated sound power.

4. DISCUSSION OF RESULTS

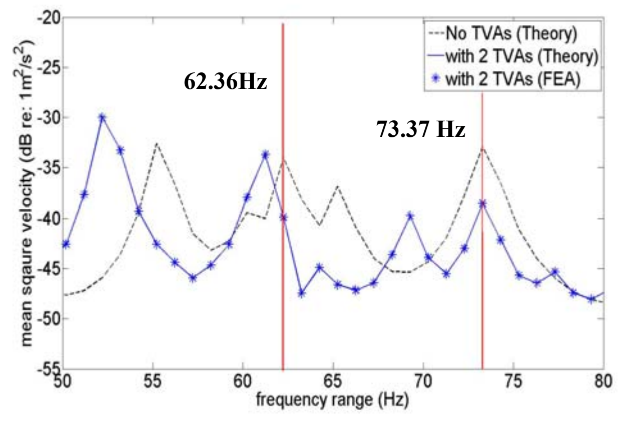
For the analyses considered here, a panel with dimensions (x,y) = (1000 mm, 1500 mm), thickness of 1 mm and loss factor of 0.01 was used for example purposes. Two locations were selected for the attachment of TVAs: (x,y) = (400 mm, 650 mm) and (500 mm, 600mm). Each TVA is intended to target resonance frequencies at 62.36 Hz and 73.37 Hz respectively. In this case, the chosen mass ratio between the total added mass of the TVAs and the panel is 15% (approximately 911.20 grams in total). This mass was equally distributed amongst the 2 TVAs. A point force of 1 N was applied at the location of (x,y) = (300 mm, 400mm). Table 1 shows the tuned vibration absorber parameters used in the analyses.

Table 1. Tuned vibration absorber parameters

	Mass (grams)	Frequency (Hz)	(x,y) Location (m)
1	455.6	62.36	(0.40, 0.65)
2	455.6	73.37	(0.50, 0.60)



(a)

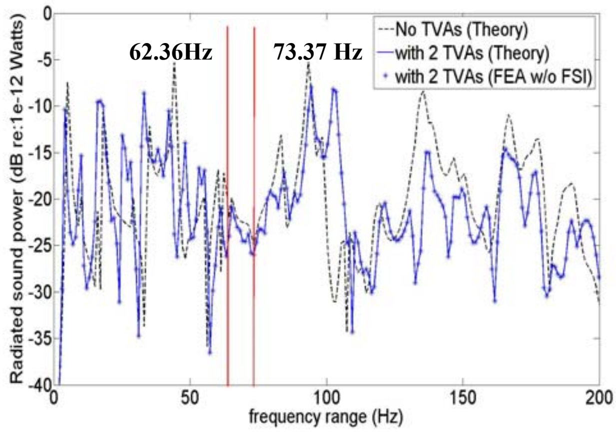


(b)

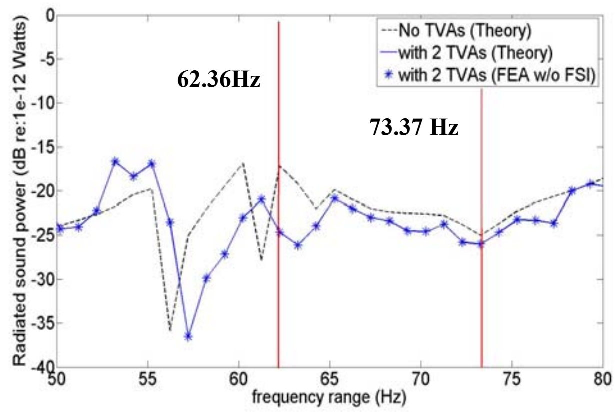
Figure 5 Mean square velocity for a panel with 2 TVAs attached, excited by a point force with an amplitude of one newton: **(a).** Comparison between theory and FEA and (**b).** Zoom - the frequency range of 50 Hz and 80 Hz of part (a).

Figure 5a shows the mean square velocity of the panel with two TVAs attached, which is calculated using the analytical formulation (solid line) and FEA (stars) with structural response only. It is observed that both the theoretical prediction and the FEA results are almost identical for the frequency range of 1 Hz to 200 Hz. Figure 5b shows the same results as Figure 5a, only over the frequency range between 50-80Hz. This figure indicates that there is no 'mode splitting' behaviour at tuning frequencies of the TVAs. However, the TVAs alter the response of the panel by effectively restraining its motion at the connection points. At these frequencies, the reduction of mean square velocity is approximately 5.8 dB and 5.6 dB respectively.

Figure 6a shows that the corresponding radiated sound power for the panel with 2 TVAs attached. The results obtained from the theoretical analysis compare reasonably well with the results from the FEA without fluid-structure interaction. Figure 6b shows that the radiated sound power is reduced near the tuning frequencies of the TVAs (62.37 Hz 73.37 Hz) by approximately 7.5 dB and 1.1 dB respectively.



(a)



(b)

Figure 6 Radiated sound power for a panel with 2 TVAs attached, excited by a point force with an amplitude of one newton: **(a).** Comparison between theory and FEA w/o FSI (without fluid-structure interaction) and **(b).** Zoom - the frequency range of 50 Hz and 80 Hz of part (a).

Figure 7 shows a comparison of the results for calculating the total radiated sound power for three cases: (1) no TVAs (Theory); (2) with 2 TVAs (FEA without fluid-structure interaction); and (3) with 2 TVAs (FEA with fluid-structure interaction). The motivation for conducting this analysis is to confirm that the calculated results for the finite element analysis without fluid-structure interaction is similar to the results for the prediction where fluid-structure interaction was included. The results compare reasonably well over the frequency range of interest. A slight deviation can be observed above 180 Hz, which is mainly due to the size of the acoustic elements. It is recommended that at least 6 elements per wavelength should be used for acoustic analyses.

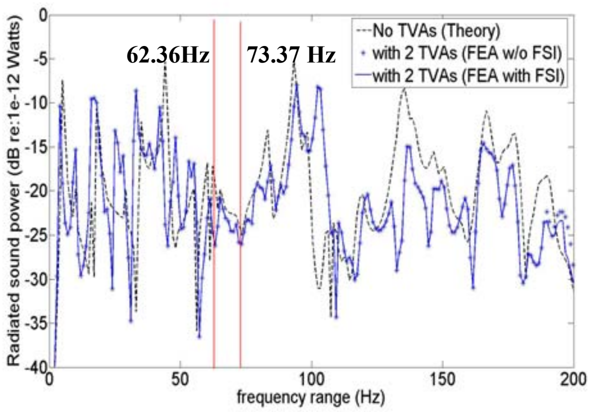


Figure 7. Radiated sound power for a panel with 2 TVAs attached, excited by a point force with an amplitude of one newton - Comparison between FEA with and without FSI.

5. SUMMARY & CONCLUSION

In this paper, a low- to mid- frequency (1Hz-200Hz) analytical model of a panel with multiple TVAs and finite element models were presented to enable the calculation of the mean square velocity and radiated sound power.

The radiated sound power was calculated by two methods. For the first method, an FEA was conducted where the model incorporated fluid-structure interaction and was used to calculate the radiated pressure. The radiated sound power was thus calculated by using a MATLAB script to calculate radiated intensity, which was then integrated intensity over a hemisphere to calculate the total radiated sound power. The second method involved the use of a finite element model without fluid-structure interaction to calculate the normal structural velocity distribution of the panel. The radiated sound power was calculated by using a MATLAB script to determine the far field radiated pressure, which was then used to calculate the intensity, and then integrated over a hemisphere.

The theoretical model presented in this paper was verified by comparing theoretical results with the results from two finite element models. A simply supported panel with two TVAs attached was used as the test case. It was showed that the results obtained from FEA compared favourably with the results from theoretical prediction.

Future work that will be investigated is to consider multiple-degree-of-freedom TVAs attached to the panel for reducing sound radiation.

ACKNOWLEDGEMENT

The first author would like to thank Karen Adams for her suggestions in preparation of the manuscript as well as Izzuddin Zaman for his suggestions on finite element modelling in this paper.

REFERENCES

Brennan, M. J. and Dayou, J. 2000, *Global control of vibration using a tunable vibration neutralizer*, Journal of Sound and Vibration, 232 (3), p. 585-600

Esteve, S. J. 2004, *Control of sound transmission into payload fairings using distributed vibration absorbers and Helmholtz resonators*, PhD Thesis Virginia Polytechnic Institute and State University

Fuller, C.R., Maillard, J. P., Mercadal, M. and Flotow, A. H. 1997, *Control of aircraft interior noise using globally detuned vibration absorbers,* Journal of Sound and Vibration, 203, p. 745-761

Fuller, C. R. and Cambou, P. 1998, *An active-passive distributed vibration absorber for vibration and sound radiation control*, The Journal Acoust. Soc. Am. 104 (3), p. 1775

Fuller, C. R., Kidner, M. R. F., Li X. And Hansen, C. H. 2004, *Active-passive heterogeneous blankets for control of vibration and sound radiation,* Proceedings of Active99, Williamsburg

Gentry, C. A., Guigou, C. and Fuller, C. R. 1997, *Smart foam for applications in passive-active noise radiation control,* Journal of Acoust. Sco. Am., 101 (4), p. 1171-1178

Grissom, M. D. 2003, *Quiet product design using optimized broadband vibration absorbers,* PhD Thesis, The Pennsylvania State University

Hansen, C. H. and Snyder, S. D. 1997, *Active control of noise and vibration,* E & FN Spoon, London

Howard, C. Q. 2008, *Transmission loss of a panel with an array of tuned vibration absorbers,* Acoustics Australia, 36, p. 98-103

Jolly, M. R. and Sun J. Q. 1996, *Passive tuned vibration absorbers for sound radiation reduction from vibrating panels,* Journal of Sound and Vibration, 191 (4), p. 577-583

Marcotte, P., Fuller, C. R. and Cambou, P. 1999, *Control of noise radiated from a plate using a distributed active vibration absorber,* Proceedings of Active 99, p. 447-456

Mead, D. J. 1990, *Plates with regular stiffening in acoustic media: vibration and radiation,* The Journal of Acoust. Soc. Am. 88, p. 391-401

Soedel, W. 1993, *Vibrations of shells and plates,* Marcel Dekker, Inc, New York

Wallace, C. E. 1972, *Radiation resistance of a rectangular panel*, The Journal of the Acoust. Soc. of Am., 51, p. 946-952

Wright, R.I. 2003, *A hierarchical noise control system using adaptable tuned vibration absorbers,* PhD Thesis, Virginia Polytechnic Institute and State University