Estimating sonar system losses due to signal spatial decorrelation

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ABSTRACT

Sonar performance modelling is often based on loss budget afforded by the standard sonar equation, which inevitably involves various idealisations and assumptions. Field performance of practical sonar systems is often observed to be below the level predicted by the conventional sonar equation based on system technical specifications. The departure in performance of practical systems from idealised systems is attributed to various extra "loss factors". These loss factors are usually combined and collectively described as "system losses". Some of the loss factors are system-related such as those associated with sound projection, reception and processing by the system. Others are caused by phenomena outside the system such as signal coherence degradation and time spreading due to reflections from targets and multi-path propagation. In this paper, we review and discuss theoretical and experimental work in assessing signal spatial de-correlation due to random environmental inhomogeneities and multipath propagation in both deep and shallow waters. Rough estimates of the resulting losses in array signal gain from conventional beamforming were given where possible.

INTRODUCTION

Sonar performance modelling is often based on loss budget afforded by the sonar equation, A decibel form of the active sonar equation can be written as (Cox 1989, Waite 2002),

$$SE = SL–2TL+TS –[(N_0+10\log B –AG) \oplus (RL_1+10\log T)] – (DT–PG) – \text{SysLoss} \quad (1)$$

Where the symbol ⊕ represents intensity summation, and

SE = signal excess,

SL = source power level,

TL = one-way transmission loss,

TS = target strength,

N_0 = noise power spectrum level,

B = receiver bandwidth,

AG = Array Gain, which is the coherent spatial processing gain of the beamformer against noise due to directivity of the receiver;

RL_1 = in-beam reverberation level over the full receiver bandwidth, normalized to 1 sec pulse, which includes reverberation reduction due to directivities of both the source and receiver beam patterns;

T = pulse duration,

DT = Detection Threshold - SNR required at the *output* of the temporal processor for certain probability of detection and false alarm;

PG = Processing Gain of the temporal processor,

SysLoss = system loss due to various idealised assumptions.

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We note that (N_0+10log *B* –*AG*) is the in-beam noise over the full receiver bandwidth; and (RL_1+10log *T*) is the in-beam reverberation level over the full receiver bandwidth.

Similarly for passive sonar,

$$SE = SL– TL – (N_0+10\log B –AG) – DT – \text{SysLoss} \quad (2)$$

Application of the sonar equation involves various idealisations and assumptions. Field performance of practical sonar systems is often observed to be below the level predicted by the conventional sonar equation based on system technical specifications. The departure in performance of practical systems from idealised systems is attributed to various extra "loss factors". These loss factors are usually combined and collectively described as "system losses". Some of the loss factors are system-related such as those associated with sound projection, reception and processing by the system. Others are caused by phenomena outside the system such as signal coherence degradation and time spreading due to reflections from targets and multi-path propagation. Therefore the system loss (SysLoss) in Eq.(1) and (2) depends on the idealised assumptions made in evaluating other terms of the sonar equation.

In this paper, we consider the losses in array gain due to environment-induced signal spatial decorrelation, which may arise from (1) deterministic multipath interference or time spreading, and (2) random spatial and temporal fluctuations in the water mass and ocean boundaries.

Wavefront curvature due to a source from a finite distance also increases the decorrelation of signals incident on a long array from broadside. This "near field" effect on array directivity has been discussed in Ziomek (1995). We do not consider wavefront curvature effect in this paper - we assume that the receiver array is sufficiently far from the source such that the Fraunhofer far-field condition as stated by Eq.(6.2.39) of Ziomek (1995) is satisfied.

ARRAY SIGNAL GAIN OF PARTIALLY CORRELATED SIGNALS

In decibel form, Array gain (AG) equals array signal gain (ASG) minus array noise gain (ANG) (Carey 1998). Spatial de-correlation of signal decreases array gain. Spatial correlation of noise may decrease (for positive correlation of noise across the array) or increase (for negative correlation of noise across the array) array gain. In this paper, we only consider the effect of signal de-correlation on array sigal gain.

In linear form, the array signal gain, G_s, may be written as (Cox 1973, Carey 1998)

$$G_s = \mathbf{u}'\mathbf{R}_s\mathbf{u} \tag{3}$$

Where $\mathbf{R}_s$ is the normalised signal cross-spectral matrix, which is a measure of the signal correlation across the array aperture.

For conventional beamforming, $\mathbf{u}$ is the steering vector of the array. For an unshaded line array of N elements steered broadside, the array signal gain becomes

$$G_s = \sum_{n=-(N-1)}^{N-1} (N-|n|)\rho(|n|d) \tag{4}$$

where d is element spacing, and ρ is the normalized spatial correlation, which is defined by the cross correlation between two complex acoustic pressure field, $p(x)$, and $p(x+\Delta x)$, separated by a distance Δx, normalized by the square root of the product of the autocorrelations of the individual signals.

$$\rho(\Delta x) \equiv \frac{\langle p^*(x)p(x+\Delta x)\rangle}{\sqrt{\langle p^*(x)p(x)\rangle\langle p^*(x+\Delta x)p(x+\Delta x)\rangle}} \tag{5}$$

Where p < > denotes ensemble average and * denotes complex conjugate.

Degradation of array gains has been studied when the spatial correlation function is assumed exponential (Cox 1973) and linear (Green 1976). Carey (1998) and Beran & McCoy (1987) used the following form for the correlation function,

$$\rho(\Delta x, L_e) = \exp[-(\Delta x / L_e)^n], \quad n = 1, 1.5, 2 \tag{6}$$

Where Δx is the separation between the two points where the acoustic pressure is measured and L_e is the correlation length where the correlation falls to 1/*e*. The correlation function is exponential when n = 1 and Gaussian when n = 2.

For fully correlated signals, ρ = 1 and the theoretical signal gain is $G_s(\rho = 1) = N^2$. When the signal is partly correlated among the array elements (ρ <1), the signal gain is less than N^2 and is given by Eq.(4). The loss of signal gain is, in dB, the ratio of signal gain to the gain when the signal is fully coherent, that is,

$$L_{SG} = 10\log_{10}\left[(1/N^2)\sum_{n=-(N-1)}^{N-1}(N-|n|)\rho(|n|d)\right] \tag{7}$$

Figure 1 shows the loss in array signal gain versus the ratio of array length over correlation length for different correlation functions. It can be seen that the loss in signal gain is about 2 dB when the array length is twice the correlation length. We also see that the loss in signal gain is not very sensitive to the exponent *n*.

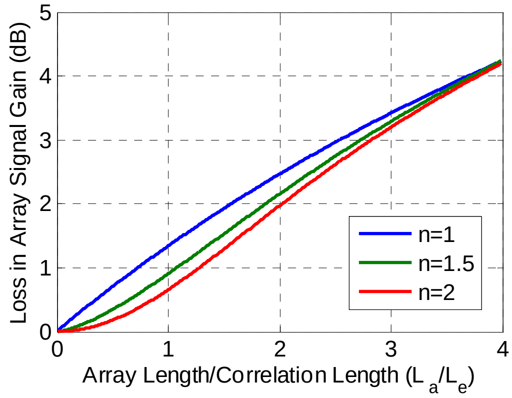

Figure 1. Loss in array signal gain vs ratio of array length over correlation length for different correlation functions (L_a = length of array, L_e = correlation length where the correlation falls to 1/*e*.)

As an example, Figure 2 shows the Array Gain in uncorrelated noise when the element spacing is half a wavelength and the correlation length is 50 wavelengths. It can be seen that increasing the array length beyond 2 correlation lengths is not worthwhile because doubling the array length to 4 correlation lengthes leads to an additional array gain of about 1 dB.

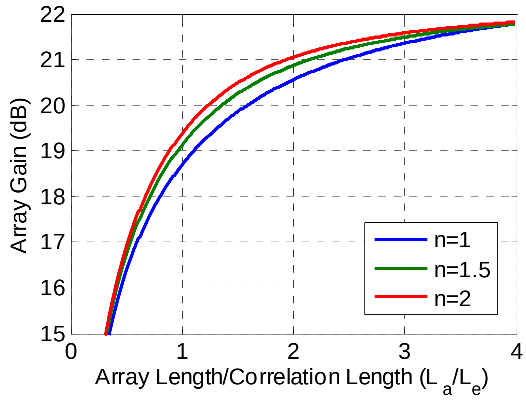

Figure 2. Array Gain in uncorrelated noise when the element spacing is half a wavelength and the correlation length is 50 wavelengths.

SIGNAL SPATIAL CORRELATION IN DEEP WATER

Signal spatial correlation depends on stochastic and deterministic spreads of the multipath energy arrival angles at the receivers (Jobst and Zabalgogeazcoa 1979). It also depends on the orientation of the array relative to the sound propagation direction. We discuss signal spatial correlations in the transverse, radial and vertical directions.

Transverse Horizontal Correlation

Besides wavefront curvature, de-correlation in the transverse direction is mainly due to stochastic scattering from random spatial inhomogeneities in water and ocean boundaries rather than deterministic interferences from multipaths. In deep water, theoretical modelling of single path propagation in an ocean with horizontal temperature fluctuations leads to the following transverse correlation function of the acoustic pressure field (Carey and Mosley 1991),

$$\rho(d_T) = \exp[-(d_T / L_T)^{3/2}] \quad (8)$$

where d_T is the transverse separation between two receivers and L_T is the transverse horizontal correlation length,

$$L_T = (E_f r)^{-2/3} f^{-5/3} \quad (9)$$

where r is the propagation range in meters, f is acoustic frequency in Hz. The temperature fluctuation coefficient E_f ranges from 3.4×10^{-17} to 13.1×10^{-17} with a mean of 4.8×10^{-17} (Carey 1998).

To obtain rough estimates of losses due to signal transverse de-correlation, we may compute the correlation length using Eq.(9) and estimate the losses from the n = 1.5 curve in Fig.1. Figure 3 shows the signal transverse correlation lengths in units of wavelength computed using Eq.(9) with the E_f coefficient in Table III of Carey (1998) for Pacific deep waters.

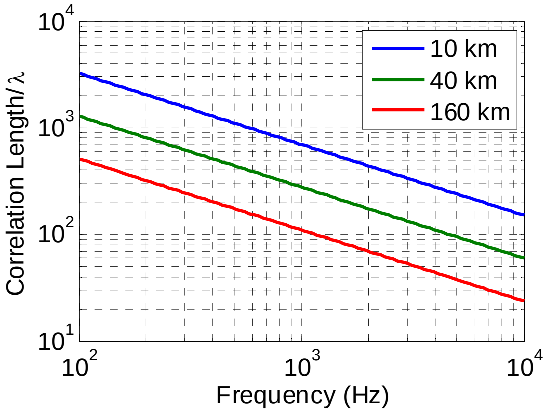

Figure 3. Signal transverse horizontal correlation lengths in deep Pacific water for different propagation ranges.

Radial Horizontal Correlation

As a typical approximation of the deep water environment, Smith (1976) considered a bi-linear sound speed channel in which the sound speed increases linearly with vertical distance on either side of an “axis” where the speed is a minimum:

$$c(z) = \begin{cases} c_0(1 + a_+ z), & z > 0 \\ c_0(1 - a_- z), & z < 0 \end{cases} \quad (10)$$

Where the gradients a_+ and a_- are positive constants. It is further assumed that all energy propgates within the angular boundaries of a limiting ray, which is the ray that vertexes at the channel edge. Energy transmitted by rays more steeply inclined than the limiting ray is ignored.

The angular energy spectra, hence the correlation functions, depend on the receivers depths. Smith (1976) gave analytical approximations for the signal radial correlations for some limiting cases.

When the source is near the sound channel axis, and the receiver near the channel’s edge, the normlaized correlation is given as,

$$\rho(d_r) = \exp(ikd_r)\left[C(u) - iS(u)\right]/u, \quad u^2 = 2\theta_L^2 (d_r / \lambda) \quad (11)$$

Where d_r is the radial separation between two receivers, $k = 2\pi/\lambda$ is the acoustic wavenumber, λ is the acoustic wavelength, $C(u)$ and $S(u)$ are the standard Fresnel integrals, θ_L is the grazing angle of the limiting ray at the receiver.

When both the source and the receiver are near the sound channel axis, the normlaized correlation is given as,

$$\rho(d_r) = \exp[ikd_r(1 + \phi^2/2)]\{1 - (1+i)\left[C(u) - iS(u)\right]\}, \quad u^2 = 2\phi^2 (d_r/\lambda) \quad (12)$$

Where ϕ is the grazing angle of the ray vertexing at the receiver when it crosses the axis.

Figure 3 of Smith (1976) shows the magnitude of the normalized radial horizontal correlations for a source near the axis and receivers at various distances from the axis.

Smith and Stern (1977, 1978) extended the study to a deep water sound speed profile with a sound speed minimum that is similar to a SOFAR channel. The source is near (less than a wavelength from) the surface and the receiver near the channel axis. The water is assumed deep enough to sustain convergence zone propagation, i.e., the sound speed at the bottom is much greater than that at the surface. Smith and Stern (1977, 1978) found that the squared magnitude of the normalized spatial correlation in the radial direction can be well-fitted by a Gaussian function. Their results can be summarized as,

$$\rho(d_r) = \exp[-(d_r / L_r)^2] \quad (13)$$

Where the radial correlatioin length L_r can be written as,

$$L_r = 0.57 \times 2^{1/2} \lambda / v_l = 0.81\lambda / v_l, \quad v_l = (c_b - c_s)/c_b \quad (14)$$

Where c_b, c_s are the sound speeds at the bottom and surface respectively.

Galkin et al (2006) analysed experimental measurements in the Mediterranean Sea for explosive sources at the axis of an approximately bi-linear channel. Correlations with different averaging times were computed to show the effect of including different multipath arrivals. In the frequency band 240–340 Hz, the correlation between receivers 300 m apart is 0.85-0.95 when only a narrow ray bundle (grazing angles 2-3^o) is included. The correlations are less than 0.2 when taking into account all multi-paths including surface and bottom reflected arrivals with much greater angular spread.

Our calculations, not included here, show that Galkin’s measurements are consistent with Smith’s (1976) analytical expressions.

Vertical Correlation

Smith’s general analytical formulation [e.g., Eqs.(11-12) of Smith 1976] may be used to estimate the signal vertical correlations in multi-path, bi-linear sound speed channels bounded by limiting rays For vertical arrays with omnidirectional hydrophones, the vertical angular structure of the multipath propagation is the major contributor to signal vertical de-correlation.

Directional receivers that select multipath arrivals in a narrower angular interval greatly enhance vertical correlation length (Galkin and Pankova 2002). Experimental data in the convergence zones of the deep Atlantic Ocean for pseudonoise signals in 0.8–1.3 kHz show large correlation coefficients (>0.7–0.8) for directional receptions separated by 250 meters in depth (Galkin and Pankova 2003). Further analysis shows that the vertical correlation length can reach hundreds or even thousands of sound wavelengths (Galkin and Pankova 2005).

We point out that whilst the use of arrays with directional receiving elements may enhance signal correlation, this will introduce another form of system loss – an "energy splitting loss" associated with the transmission loss term in the sonar equation because some signal energy outside the selected angular range is excluded from contributing to signal level.

SIGNAL SPATIAL CORRELATION IN SHALLOW WATER

In shallow water, signal correlation is the result of two competing mechanisms: random volume and boundary scattering and high-angle energy stripping due to lossy boundaries. The former decreases signal correlation with range whereas the later generally increases signal correlation with range. In general, signal correlation is low at near field because the higher order modes scatter from the water mass and rough boundaries. Then correlation may increase as higher-angle energies are being stripped away by the lossy boundaries, reducing the angular spread of the signal. In the far field, correlation decreases with range again as the remaining low-order modes scatter from water-mass fluctuations (Zhu and Guan 1992). In downward refracting propagation, the range dependence is complicated. The correlation is greatest if both the source and receiver are below the thermocline because the field is then dominated by the lower order modes.

Transverse Horizontal Correlation

Wille and Thiele (1971) measured the transverse horizontal coherence in shallow (65 m) water using explosive signals. The environment was isothermal mixed layer (upward refracting). The sea surface was rough with characteristic wave-height much greater than the roughness of the sea bottom. The absence of the temperature gradient meant minimal internal wave activity and that signal decorrelation was mainly due to reflections from the rough sea surfaces. Wille and Thiele (1971) used a Gaussian distributed angular power directivity to yield a Gaussian spatial correlation function

$$\rho(d_T) = \exp\left[-(2\pi\, d_T \sigma_\theta / \lambda)^2 / 2\right] \qquad (15)$$

where d_T is the transverse separation between two receivers, λ is the acoustic wavelength, σ_θ is the standard deviation (angular spread). The correlation length L_T where the correlation falls to 1/e is given by

$$L_T = \lambda / (\pi\, \sigma_\theta \sqrt{2}) \qquad (16)$$

Carey et al (2002) measured array signal gain at three shallow water sites with variable downward refracting conditions. The loss in signal gain was used to estimate that between 300 -400 Hz, the horizontal coherence lengths are about 30 wavelengths to ranges of 40 km.

Measurements in 200-300 meters of coastal water of the Barents Sea show that for frequencies below 300 Hz, the transverse correlation length was greater than 2 km and only weakly depends on the range up to 110 km to the sound source. For frequencies 350 to 500 Hz, it gradually decreases with increasing range, which has been attributed to the effect of short-period internal waves (Galkin, Popov & Simakina 2004). Galkin et al (2004)'s correlation lengths are much greater than those in Wille and Thiele (1971), possibility because both the source and receiver depths are near the channel axis of the sound speed profile.

Figure 4 shows experimental results in shallow water with isothermal (Wille & Thiele, 1971) and downward refracting profiles (Carey etal 2002). The solid curve is a simple fit to the data of Wille & Thiele (1971).

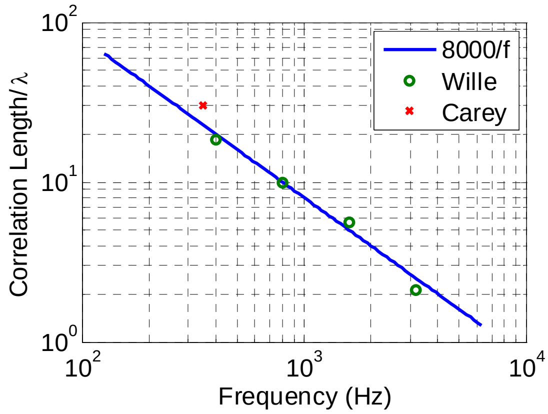

Figure 4. Signal transverse horizontal correlation lengths in shallow water with isothermal (Wille & Thiele 1971) (circle) and downward refracting (Carey 1998, Carey etal 2002) (cross) profiles.

In the absence of sufficient data, the following process may be used to obtain a rough estimate of the loss due to signal transverse horizontal de-correlation:

- In shallow water with mixed layer profile, estimate the correlation length using the blue curve in Fig.4 and estimate the loss from the n = 2 curve in Fig.1. This probably provides an upper bound to the loss because the blue curve is based on Wille & Thiele (1971)'s data obtained under heavy sea state 4.

- In shallow water with downward refraction profile, estimate the correlation length using Carey's data (Carey 1998, Carey etal 2002) in Fig.4, interpolate based on $(\text{frequency})^{-1}$ variation if necessary and estimate the loss from the n = 2 curve in Fig.1.

Radial Horizontal Correlation

General analytical expressions for signal radial correlations were derived based on angular energy spectra (Smith 1976) and normal mode considerations (Wang & Zhang 1992).

Smith (1976) gave the following expression for the radial horizontal correlation in an isospeed shallow water channel,

$$\rho(d_r) = \left[1 + (k d_r \bar{\theta}^2 / 2)^2\right]^{-1/4}, \qquad \bar{\theta}^2 = 2H / br \qquad (17)$$

Where $k = 2\pi/\lambda$ is the acoustic wavenumber, $\bar{\theta}$ is the characteristic angular spread, H is the water depth, r is the horizontal range, and b is a measure of boundary absorptivity (The intensity of a ray striking the boundaries at a grazing angle θ is attenuated by a factor exp($-b\theta$) after one bounce from each of the two boundaries, i.e., one ray cycle distance.)

We can see that the radial correlation increases with range and the rate of boundary absorption as lossy boundaries strip away high-grazing angle multipath energies. It also decreases with water depths because greater water depths accommodate greater spread of propagation angles.

Jones et al gave slightly different expressions for isospeed channels also based on angular energy spectra arguments (Jones, Duncan, & Maggi 2007).

Vertical Correlation

Analytical expressions for signal vertical correlations were given based on angular energy spectra and normal mode considerations (Smith 1976, Wang & Zhang 1992). The expressions lead to Gaussian correlation functions for isospeed shallow water channels.

Smith (1976) gave the following expression for the vertical correlation in an isospeed shallow water channel,

$$\rho(d_v) = \exp\left[-(kd_v\bar{\theta}/2)^2\right], \quad \bar{\theta} = [2H/(br)]^{1/2} \tag{18}$$

Where d_v is the vertical separation between two receivers. The vertical correlation length, where the correlation falls to 1/*e*, is then,

$$L_v = (\lambda/\pi)[br/(2H)]^{1/2} \tag{19}$$

We can see that the vertical correlation lengths increase with the square root of range and the rate of boundary absorption as lossy boundaries strip away high-grazing angle multipath energies. They decrease with water depths because greater water depths accommodate greater spread of propagation angles.

Also based on angular energy spectra arguments, Jones, Duncan, & Maggi (2007) gave slightly different expressions for the vertical correlation length in isospeed channels.

Measurements in 200-300 meters of coastal water in the Barents Sea with source and receiver depths near the sound speed minimum show that vertical correlations increase with range up to 200 km in the frequency band 240 to 340 Hz.

At lower frequencies, vertical signal correlation was measured by transmitting CW pulses of 107 Hz and 240 Hz from a source near the bottom to a vertical line array 13.82 km away in a downward refracting shallow water environment (Sazontov, Matveyev & Vdovicheva 2002). The data in Table I & Fig.12 of Sazontov, Matveyev & Vdovicheva (2002) seem to show that (1) the correlation lengths are much greater than two wavelengths; and (2) decorrelation increases more than linearly with frequency.

Yang (2007) measured vertical signal correlation by transmitting 0.1s duration LFM pulses of 400 Hz bandwidth centred around 1200 Hz and correlating the arrivals received on a vertical line array. The environment was coastal waters of 100 m depth with a downward refracting (summer) profile. The source was 4 m above the bottom and 33 receivers, separated by 0.5 m (which is 0.4 wavelength at 1200 Hz), were between 50 – 66 m. The source to receiver range was about 10 km. It was found that the measured vertical correlation among the adjacent 8 receivers can be fitted with a Gaussian function,

$$\rho(d_v) = \exp[-(d_v/L_v)^2] \tag{20}$$

where d_v is the vertical separation of the receivers, and the vertical correlation length L_v is about 2 wavelengths. (In passing, we note that based on Eq.(18), a vertical correlation length L_V of 2 wavelengths corresponds to an angular spread of 13 degrees, which matches well with the angular spread shown in Fig.3b of Yang (2007).

As shown in Fig.5 of Yang (2007), the Gaussian function in Eq.(20) under-estimates the signal correlation beyond the adjacent 8 receivers, leading to under-estimation of the signal gain shown by the solid line in Fig.6a of Yang (2007).

We use the following modified Gaussian correlation function to account for the residual correlations beyond the nearby 8 receivers;

$$\rho(d_v) = \begin{cases} \exp[-(d_v/L_v)^2], & |d_v| \le L_v \\ \exp(-1) \approx 0.37, & |d_v| > L_v \end{cases} \tag{21}$$

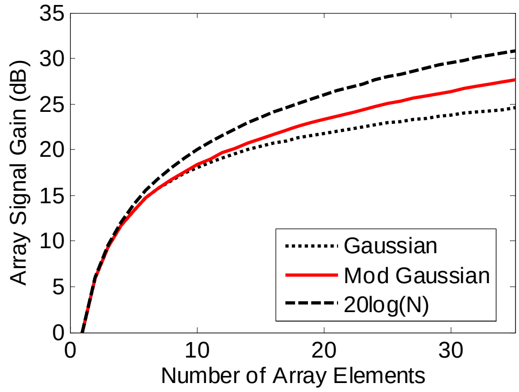

Figure 5. Signal gain versus the number of array elements for a vertical line array: signals with full correlation (dashed curve), Gaussian correlation (dotted curve), modified Gaussian correlation (red curve).

Figure 5 shows computed signal gain as a function of the number array elements and can be directly compared with Fig.6a of Yang (2007). The dashed curve is for fully correlated signals; the dotted curve is for signals with the Gaussian correlation in Eq.(20), and the solid curve is for signals with the modified Gaussian correlation in Eq.(21). The dashed and dotted curves show the same results as the two corresponding curves in Fig.6a of Yang (2007). The solid curve, computed using the modified Gaussian correlation in Eq.(21), fits well the measured signal gain data in Fig.6a of Yang.

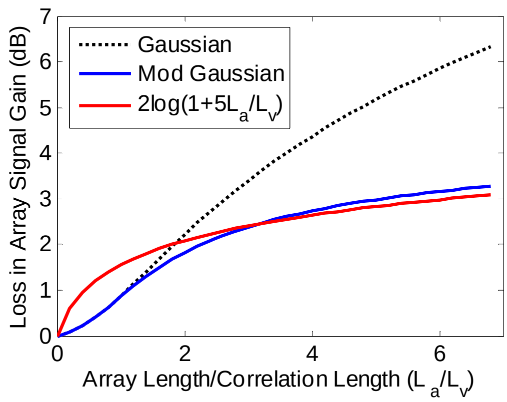

Figure 6. Loss in signal gain vs ratio of array length over correlation length for a vertical line array: Gaussian correlation (dotted curve); modified Gaussian correlation (blue curve); and a simple fit 2log(1+5L_a/L_v) (red curve).

Figure 6 shows the corresponding loss in signal gain as a function of the ratio of array length L_a over signal vertical correlation length L_v. The red solid curve is a simple fit to the results using the modified Gaussian correlation in Eq.(21). The result may be used to quickly assess array signal gain degradation in situations where the signal correlation length are approximately known or can be estimated.

Besides Yang's (2007) work, there is little other experimental data to indicate if the Gaussian form or the modified Gaussian form is a closer approximation to signal vertical correlation in shallow water, although the data at low frequencies in Sazontov, Matveyev & Vdovicheva (2002) support the assumption that correlation does maintain a certain value after the initial drop with sensor separation distance. The blue curve or the simple fit $2\log(1+5L_a/L_v)$ in Fig.6 may be used as a reasonable estimate of signal gain loss in shallow downward refracting environments for vertical line arrays. For two or three dimensional arrays, the estimate constitutes the loss component in array gain degradation due to signal vertical decorrelation, which can be combined with the loss components due to signal decorrelation in other dimensions.

SUMMARY

We reviewed and discussed theoretical and experimental work in assessing signal spatial de-correlation due to stochastic inhomogeneity and multipath propagation in both deep and shallow water environments. Rough estimates of the resulting losses in array signal gain from conventional beamforming were given where possible.

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