Enhancements to a tool for underwater acoustic sensitivity analysis, and relative importance of uncertainties in environmental parameters

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ABSTRACT

Current sonar prediction models for an active sonar scenario give outputs such as probability of detection. However, the inputs to the models can be very uncertain, and it would be desirable to present measures of *uncertainty* in the output, and of *sensitivity* of the output to uncertainty in each of the inputs. In a previous paper, a tool was developed to predict uncertainty and sensitivity in acoustic transmission loss (*TL*) to uncertainty in sound speed profile (*SSP*) and bottom loss (*BL*). For the Malta plateau environment, it was found that *SSP* uncertainty was more important than *BL* uncertainty at some ranges, and vice-versa at other ranges. In this paper, an option has been provided to evaluate the sensitivity measure directly and more accurately than the (albeit faster) Monte-Carlo method used in the previous paper. The tool has been extended to compute probability of detection (P_d) and its uncertainty and sensitivity. For the Malta plateau environment and for some input parameters, it was found that SSP uncertainty was the most important parameter in P_d sensitivity at all ranges greater than 3.2 km, whereas for *TL*, *BL* sensitivity was most important at some ranges greater than 3.2 km. An explanation for these results is suggested.

INTRODUCTION

Current sonar performance prediction models give outputs such as “detection range of the day” versus azimuth, and probability of detection versus range. However, the inputs to a sonar performance prediction model, such as sound speed profile (*SSP*) and bottom loss (*BL*) can be very uncertain due to incomplete measurements, or spatial or temporal fluctuations. It would be desirable to present to the operator some measure of prediction uncertainty, and of sensitivity to uncertainties in each of the inputs. In the sensitivity literature, the analysis of output uncertainty due to *all* the input parameters is *uncertainty analysis* (UA); and the analysis of output uncertainty due to *each* input parameter (or the “importance” of each input parameter) is *sensitivity analysis* (SA). The main steps of UA and SA are:

(1) generate a Monte-Carlo ensemble of uncertain inputs according to a specified multivariate distribution

(2) propagate the ensemble of inputs through the input-output model to yield an ensemble of outputs

(3) estimate the uncertainty in the output from its ensemble and

(4) estimate the sensitivity of the output to uncertainty in each individual input.

In a previous paper (Sweet and Jones 2008), an existing UA/SA modelling package – *Algorithms for Stochastic Sensitivity Analysis* (ASSA, by Jansen 2005) – was adapted to develop a tool to use uncertainty models of *SSP* and *BL* to predict uncertainty and sensitivity in acoustic transmission loss (*TL*). The uncertainty and sensitivity measures were the total variance (*VTOT*) and the “top marginal variance” (*TMV*) respectively of *TL* – the latter is the expected reduction in variance which would occur if the input of interest were known perfectly. We refer to this tool as the Underwater Acoustic Sensitivity Tool, version 1 (UAST v.1).

Results (Sweet and Jones 2008) were obtained for the Malta Plateau environment (Dosso *et al* 2007), and it was found that *SSP* uncertainty was more important than *BL* uncertainty at some ranges, and *vice-versa* at other ranges.

This paper describes two main extensions to UAST v.1, and discusses results obtained from them. The first provides an option to evaluate the expression for *TMV* directly, and more accurately; UAST v.1 used a Monte-Carlo method. Secondly, the tool has been extended to compute probability of detection (P_d) for an active sonar scenario, and its uncertainty and sensitivity. We refer to the updated version of the UAST as UAST v.2.

P_d results were obtained for the Malta Plateau for assumed signal and noise levels, and it was found that for certain values of noise level, *SSP* was more important than *BL* at all ranges greater than 3.2 km, whereas for *TL*, *BL* sensitivity was most important at some ranges greater than 3.2 km. An explanation of these results is suggested.

The paper is organized as follows. In the next section, the ASSA package and UAST v.1 are reviewed. Some uncertainty and sensitivity results for *TL* obtained using UAST v.1 are then reviewed and discussed. In the following two sections, the two extensions to UAST v.1 are described: direct evaluation of *TMV*, and computation of active sonar P_d and its uncertainty and sensitivity. P_d results are then presented and compared with the *TL* results, and an explanation offered for the reversal of importance of input parameters between the *TL* and P_d plots at some ranges. Finally, some conclusions are drawn and proposals for future work are given.

ASSA AND UAST V.1

ASSA

ASSA – Algorithms for Stochastic Sensitivity Analysis (Jansen 2005) – is a library of C-language algorithms which can be used to perform UA and SA. They fall into four groups, corresponding to the four main steps of UA and SA listed above in the Introduction:

- input generators;
- input-output models;
- uncertainty analysis; and
- sensitivity analysis.

Input Generators

The input generators provide a set of multidimensional samples (one dimension per input) whose distributions and correlations are as specified by the user. ASSA provides two basic generators – uniform and multinormal. For other distributions, ASSA allows the user to carry out two steps. Here, m is the number of inputs.

- Draw a set of m-dimensional samples uniformly from the m-dimensional hypercube, with the desired correlation matrix
- Transform these samples to variables with the desired distribution

Input-Output Models

Only two very simple models, suitable for software development, are available (Jansen 2005). The user is expected to integrate their own model into their analyses, which includes computation of the output ensemble, say y, from the input ensemble.

Uncertainty Analysis

Variance based methods are used for both uncertainty and sensitivity analysis. ASSA uses the total output variance due to all input parameters

$$VTOT = \mathrm{Var}(y)\ . \qquad (1)$$

Sensitivity Analysis

ASSA uses two measures of sensitivity to input parameters, the *top marginal variance (TMV)* and the *bottom marginal variance* (Jansen et al. 1994). The former is the most useful, and we review that here.

The TMV due to a group S of inputs is the expected variance reduction that would occur if one knew S perfectly. For example, suppose there are two input parameters a_1 and a_2 and one wishes to find the TMV of the output y due to a_1. Let the limits of a_1 be $a_{1\min}$ and $a_{1\max}$. Fix $a_1 = a_{1\min}$, compute y and its (reduced) variance $\mathrm{Var}_1(y)$ for an ensemble of values of a_2. Increment a_1 a little and repeat, yielding $\mathrm{Var}_2(y)$. Continue until $a_1 = a_{1\max}$, yielding say $\mathrm{Var}_K(y)$. Average all the $\mathrm{Var}_j(y)$, yielding the average reduced variance, say $\mathrm{Var}_{red}(y)$. Then the average reduction, or the TMV, is $\mathrm{Var}(y) - \mathrm{Var}_{red}(y)$.

It can be shown (Saltelli *et* al 2004) that the TMV due to S, denoted by TMV_S, is given by

$$TMV_S = \mathrm{Var}[\mathrm{E}(y|S)] \qquad (2)$$

where E denotes expectation. ASSA's measure of sensitivity is actually the ratio $TMV_S / VTOT$, the *relative TMV*.

ASSA provides two methods of computing TMV and BMV, a *regression-based* method, which fits a linear model to the output data, and a *regression-free* method, which makes no assumptions about the form of the model. We only consider the latter here.

Computation of $TMV_S/VTOT$ using the resampling method

Equation (2) computes TMV_S exactly. ASSA does not compute (2) directly, but uses Saltelli *et al*'s (2000) *resampling-based* method, a Monte-Carlo method which provides an estimate of TMV_S in fewer operations. They use the following procedure:

Algorithm 1 (Jansen 2005)

1. Construct *two* independent ensembles of inputs $\mathbf{X}_1$ and $\mathbf{X}_2$. $\mathbf{X}_1$ and $\mathbf{X}_2$ can be represented as matrices with N rows and m columns, where N is the number of samples in each ensemble and m is the number of inputs.
2. Let the column indices of the inputs in S be $\{i_1, \ldots i_p\}$. Note that $p \leq m$. Set columns $i_1, \ldots i_p$ of $\mathbf{X}_2$ equal to columns $i_1, \ldots i_p$ respectively of $\mathbf{X}_1$.
3. Using the chosen model, compute two output ensembles $\mathbf{y}_1$ and $\mathbf{y}_2$ from $\mathbf{X}_1$ and $\mathbf{X}_2$ respectively.
4. Compute the two sample variances v_1 and v_2 from $\mathbf{y}_1$ and $\mathbf{y}_2$ respectively. Estimate $VTOT = \mathrm{Var}(y)$ by the geometric mean of v_1 and v_2, that is $VTOT = \sqrt{v_1 v_2}$.
5. Let $\mathrm{corr}(\mathbf{y}_1, \mathbf{y}_2)$ be the Pearson correlation coefficient of $\mathbf{y}_1$ and $\mathbf{y}_2$ (Parratt 1961). It can be shown (Saltelli et al. 2000) that $\mathrm{corr}(\mathbf{y}_1, \mathbf{y}_2)$ is an estimate of the relative TMV:

$$TMV_S/VTOT \cong \mathrm{corr}(\mathbf{y}_1, \mathbf{y}_2). \qquad (3)$$

UAST v.1

UAST v.1 adapts the ASSA methodology to the estimation of uncertainty and sensitivity of transmission loss (TL) in the underwater acoustic channel to uncertainty in sound speed profile (SSP) and bottom loss (BL). TL is an important parameter in the estimation of sonar performace. It is defined by

$$TL = -20 \log_{10}(p / p_1) \qquad (4)$$

where p_1 and p are the rms sound pressure at 1m from the transmitter and at the receiver respectively. We review how the four main steps of UA/SA are implemented in UAST v.1.

Input Generators

The two main parameters that affect TL are SSP and BL. We neglect the effect of surface absorption loss in this paper.

SSP uncertainty model

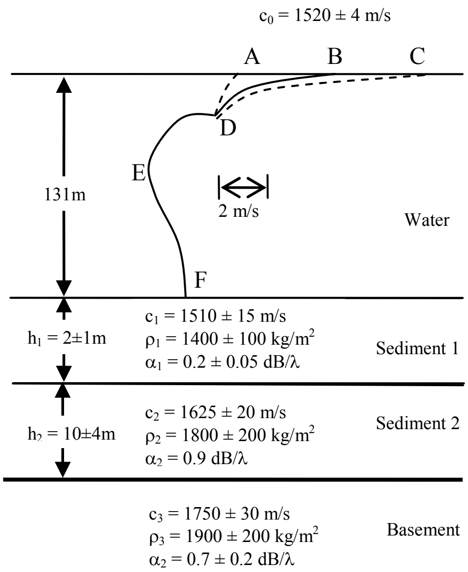
In UAST v.1, the SSP uncertainty model of Dosso *et al* (2007) is used – a schematic diagram is given in Figure 1. The SSP uncertainty (indicated by the dotted lines) is taken to represent variability due to surface heating/cooling and wind mixing, with the effects decaying exponentially with depth over the top 30m, as shown in Figure 1. This variability is represented in terms of the standard deviation of the surface sound speed, which is assumed to have a Gaussian distribution. Mathematically, the variation of the perturbation with depth can be expressed as

$$\Delta c_z = \Delta c_0 \exp(-\gamma z) \tag{5}$$

where z is the depth, Δc_z is the perturbation at depth z, Δc_0 is the perturbation at the surface, and γ is the rate of exponential decay. So the perturbed *SSP* is given by

$$c_z = \tilde{c}_z + \Delta c_z \tag{6}$$

where $\tilde{c}_z$ is the unperturbed *SSP*. Note from (5) and (6) that the perturbed *SSP* can be expressed in terms of a single uncertain parameter, viz. Δc_0. So an ensemble of uncertain *SSP*s can be generated from a Gaussian ensemble of Δc_0's using (5) and (6).



Source: (Dosso *et al* 2007)

Figure 1. Sound speed profile, bottom parameters and associated uncertainties for the Malta Plateau environment
Key points in SSP (depth, sound speed):
A: (0, 1516); B: (0, 1520); C: (0, 1524)
D: (30, 1515); E: (64, 1511); F: (131, 1512)

BL uncertainty model

The bottom can be characterized by *BL* versus grazing angle (β) curves. Jones *et al* (2008) have shown that for many bottom types, these curves may be characterized by a *single parameter*, viz the slope, g, of the *BL* versus β curve at low grazing angles. They show that the magnitude of *BL* can be approximated by

$$BL \cong g\beta \tag{7}$$

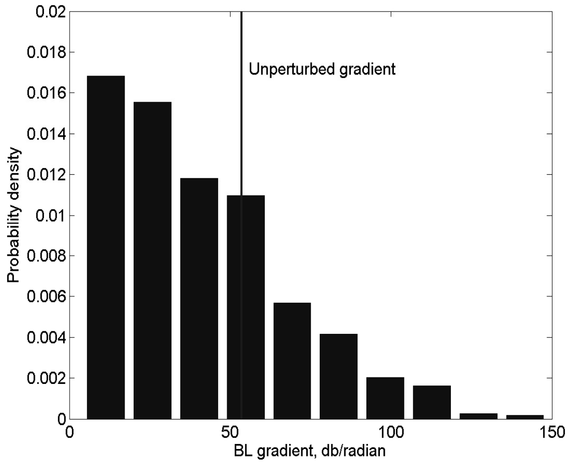
and that the reflection phase shift, φ, can be approximated by

$$\varphi \cong -\pi + \pi g\beta / 6, \quad \beta \leq 6/g, \tag{8a}$$
$$\cong 0 \quad \text{otherwise.} \tag{8b}$$

So if the distribution of g is known, an ensemble of uncertain complex bottom reflection coefficients (*BRC*s) can be generated using (7) and (8).

In UAST v.1, the distribution of g is estimated using Dosso *et al*'s (2007) uncertainty model for 11 geoacoustic parameters such as compressional sound speed, layer thickness, etc. (see Figure 1). The uncertainties in these parameters are modelled in Dosso et al. (2007) by Gaussian distributions, with unrealistic samples (such as negative thicknesses) being excluded.

In UAST v.1, a large set of samples of these parameters (varying all parameters) is generated, assuming that the perturbations are independent. For each sample of the 11 parameters, the *BL* versus β curve is calculated using a multilayer reflection coefficient formula (Bartel, D.W., private communication 2007). The bottom loss slope g is then estimated by fitting a straight line to curve at small β, specifically the interval from 0° to 10°. The distribution of the g's is then calculated, and is shown in Figure 2.



Source: (Sweet and Jones 2008)

Figure 2. Probability density function (PDF) of bottom loss gradients derived from bottom parameters and uncertainties of Figure 1

Note that this PDF is highly skewed towards the lower bottom loss slopes. The unperturbed gradient is also indicated on Figure 2. Note that the uncertainty model of Figure 1 generates more samples of g below the unperturbed value than above.

Input-output Model

In UAST v.1, the inputs are the *SSP*, the complex *BRC* curve, the absorption coefficient and the source and receiver positions. The latter three inputs are assumed known. The output is *TL*, which can be computed using a variety of models. Porter's (2008) *Acoustics Toolbox* contains a number of codes, including BELLHOP (ray-tracing model) and KRAKENC (modal model). UAST v.1 uses BELLHOP because of its speed of execution and the fact that it gives accurate results at the chosen frequency of 1200 Hz.

The procedure for generating an ensemble of outputs for UA and SA is thus:

1. Generate ensembles of uncertain *SSP*s and *BRC*s as described above.

2. Input these ensembles (together with known parameters) to BELLHOP to yield an ensemble of *TL*s.

Uncertainty and Sensitivity Analyses

We estimate *VTOT* as the measure of *TL* uncertainty, and TMV_{SSP} */VTOT* and TMV_{BL} */VTOT* as the measures of *TL* sensitivity to uncertainties in *SSP* and *BL* respectively. These are estimated using Algorithm 1. Some details of the application of Algorithm 1 to *TL* are as follows:

Step 1. $\mathbf{X}_1$ and $\mathbf{X}_2$ are independent ensembles of the pairs $[\Delta c_0, g]$

Step 2. The set S is either Δc_0 or g. For sensitivity to Δc_0, set the Δc_0's in $\mathbf{X}_2$ equal to those in $\mathbf{X}_1$. For sensitivity to g, set the g's in $\mathbf{X}_2$ equal to those in $\mathbf{X}_1$

Step 3. For each pair $[\Delta c_0, F]$ in the independent ensembles, compute the *SSP* using (5) and (6), and the *BL* curve using (7) and (8). This generates two ensembles of *SSP*s and *BL*s. Generate corresponding ensembles of *TL*s, say $\mathbf{TL}_1$ and $\mathbf{TL}_2$, using BELLHOP. These correspond to $\mathbf{y}_1$ and $\mathbf{y}_2$ in Algorithm 1.

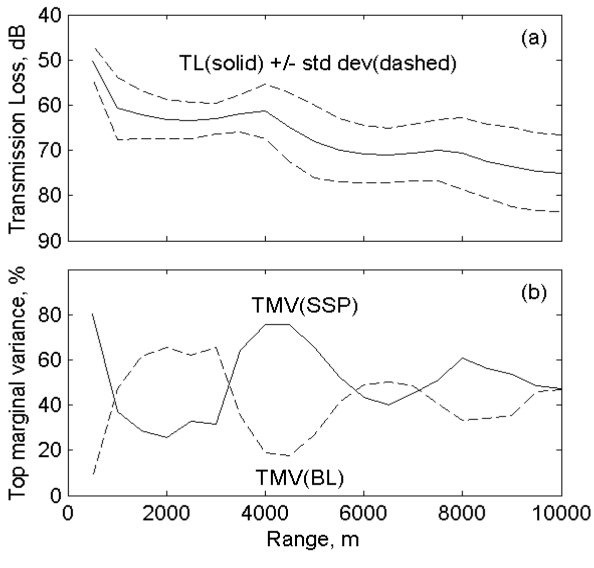
Step 4. Compute $v_1 = \mathrm{Var}[\mathbf{TL}_1]$, $v_2 = \mathrm{Var}[\mathbf{TL}_2]$ and $VTOT = \sqrt{v_1 v_2}$.

Step 5. If $S = \Delta c_0$ (in Step 2), then $TMV_{SSP}/VTOT = \mathrm{corr}(\mathbf{TL}_1, \mathbf{TL}_2)$, otherwise $S = g$ and $TMV_{BL}/VTOT = \mathrm{corr}(\mathbf{TL}_1, \mathbf{TL}_2)$

EXAMPLE UNCERTAINTY/SENSITIVITY RESULTS

We review the results presented in (Sweet and Jones 2008) for the environment of the Malta Plateau (Dosso *et al* 2007). Figure 1 illustrates the environment, its parameters and associated uncertainties (standard deviations of Gaussian distributions). The source and receiver depths are both 15m.

The UA results for this environment are shown in Figure 3, sub-plot (a). The solid curve shows the mean ensemble incoherent *TL* versus range. Note the peaks at 4km and 8km. The upper dashed curves show plots of mean $TL\pm\sqrt{VTOT}$, i.e. mean *TL* ± one standard deviation. The standard deviations vary between 4 dB and 8.5 dB.



Source: (Sweet and Jones 2008)

Figure 3. (a) UA and (b) SA of *TL* results for Malta Plateau environment using resampling method

The SA results for this environment are shown in Figure 3, sub-plot (b). The solid and dashed curves show the relative *TMV*s due to *SSP* and *BL* uncertainty respectively – we denote these by *TMV*(*SSP*) and *TMV*(*BL*). They are plotted as percentages, i.e. 100·*TMV*/*VTOT*. Note the peaks in *TMV*(*SSP*) and troughs in TMV(*BL*) at 500m, 4km and 8km. Note also the troughs in *TMV*(*SSP*) and peaks in *TMV*(*BL*) at 2km and 6.5km.

Discussion of results

Figure 4 shows a range-depth plot of unperturbed *TL*. For a slice through the plot at 15m (the receiver depth), there are peaks at ranges 4 km and 8km, and these correspond to the peaks in *TL* (Figure 3(a)).

Figure 4 also shows that *TL* changes rapidly with position at depth 15m and ranges 500m, 4km and 8km. One effect of perturbing the *SSP* is to "shift" the *TL* pattern, so at these ranges, *TL* would be expected to be sensitive to *SSP* changes.

Figure 4 shows that *TL* changes only slowly with position at depth 15m and ranges 2km and 6.5km, hence *TL* would be expected to be relatively insensitive to *SSP* changes. Conversely *TL* would be expected to be relatively more sensitive to *BL* changes at these ranges.

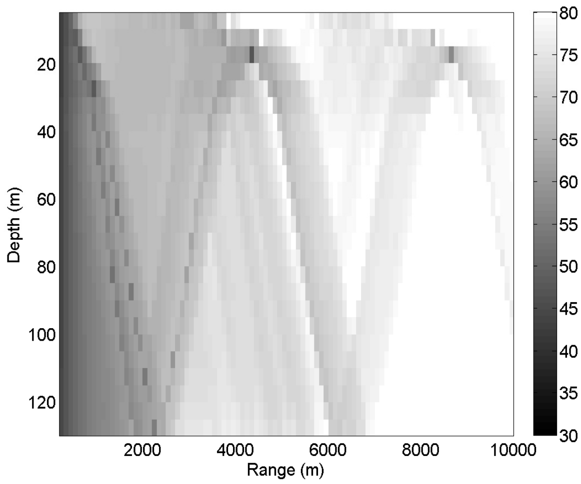


Figure 4. Unperturbed *TL* versus range and depth field for Dosso environment. Greyscale is in decibels.

DIRECT EVALUATION OF TMV

Evaluation of TMV by Algorithm 1 is fast, but does not provide the exact value. It yields an approximation that asymptotically converges to the exact value as the number of samples in the ensembles of $\mathbf{y}_1$ and $\mathbf{y}_2$ increases. For one input-output model (Simakov 2009), about 50,000 samples were required for convergence with two uncertain parameters, and the number of samples required increases rapidly as the number of parameters increases.

We recall the exact expression for *TMV* due to a set of parameters S in equation (2).

We consider the case where there are only two uncertain parameters, say a_1 and a_2. Let their probability density functions be $P_1(a_1)$ and $P_2(a_2)$ respectively, and assume that a_1 and a_2 are independent. We evaluate (2) for $S = \{a_1\}$. For any value of a_1,

$$\mathrm{E}(y|S) = \mathrm{E}(y|a_1) = \int_{-\infty}^{\infty} P_2(a_2)\, y(a_1, a_2)\, da_2 = c(a_1), \text{ say} \quad (9)$$

$$\text{Let } \bar{c} = \mathrm{E}(c) = \int_{-\infty}^{\infty} P_1(a_1)\, c(a_1)\, da_1 \,. \quad (10)$$

Note that $\bar{c} = \mathrm{E}(y) \equiv \bar{y}$. Inserting (9) into (2), we have the top marginal variance due to a_1 :

$$TMV(a_1) = \int_{-\infty}^{\infty} P_1(a_1)\,(c(a_1) - \bar{c})^2\, da_1 \qquad (11)$$

The exact expression for the total variance is

$$VTOT = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_1(a_1)\,P_2(a_2)\,(y(a_1,a_2) - \bar{y})^2\, da_1 da_2 \qquad (12)$$

and the relative *TMV* is

$$RTMV(a_1) = TMV(a_1)\ /\ VTOT\ . \qquad (13)$$

Eqs.(9)-(13) can be used to evaluate *VTOT* and $RTMV(a_1)$ directly. For our underwater acoustic UA/SA, $a_1 = \Delta c_0$, P_1 is Gaussian, $a_2 = g$, P_2 is the distribution of g as estimated above (Figure 2), and $y = TL$. $RTMV(a_2)$ can be evaluated similarly (by interchanging a_1 and a_2, and P_1 and P_2, in (9)-(13). The actual numerical steps in evaluating (9)-(13) are as follows:

- Define a grid of values of a_1 and a_2 covering the intervals of support of P_1 and P_2 respectively, or if either interval is infinite, the interval in which the probability is more than some small threshold.
- For each value of a_1, evaluate (9) numerically, yielding $c(a_1)$
- Evaluate (10) numerically, yielding $\bar{c} = \bar{y}$
- Evaluate (11) numerically, yielding $TMV(a_1)$
- For each value of a_2, evaluate

  $$t(a_2) \equiv \int_{-\infty}^{\infty} P_1(a_1)\,(y(a_1,a_2) - \bar{y})^2\, da_1 \text{ numerically}$$

- Evaluate $VTOT = \int_{-\infty}^{\infty} P_2(a_2)\,t(a_2)\,da_2$ numerically
- $RTMV(a_1) = TMV(a_1)\ /\ VTOT$.

Figure 5 shows *TL*, its uncertainty range (*TL* ± one standard deviation) and its top marginal variances versus range. The curves in Figures 3 and 5 are very similar, but not the same. Parts (a) of both figures are virtually indistinguishable by eye, except at range 500m, where a singularity in integral (12) at very high *SNR* caused a “not-a-number” to be generated in the direct *TMV* computation. In parts (b) of the figures (the *TMVs*) the peaks and troughs of the curves are in about the same places. The differences in *TMV*s between Figures 3 and 5 generally varied from zero to 10%. The differences can be attributed to the approximation used in the resampling method (Algorithm 1).

PROBABILITY OF DETECTION

An important measure of sonar performance is probability of detection. As well as *TL* it depends on several other parameters such as source level, noise level, receiver directivity index and target strength - the ratio of the echo intensity 1m from the target to the incident intensity.

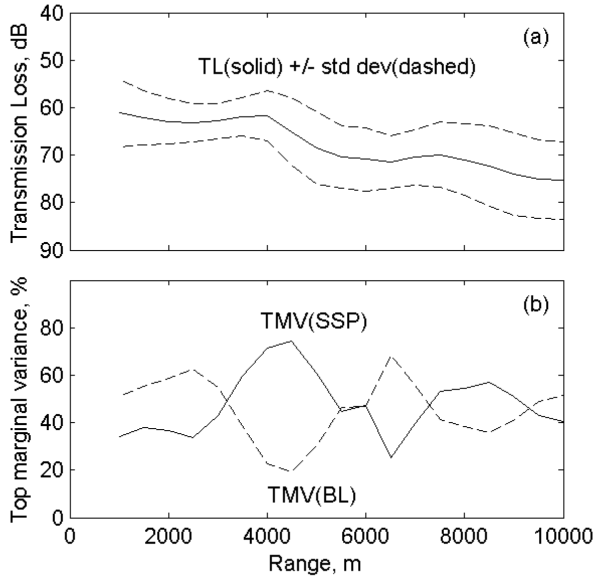


Figure 5. (a) UA and (b) SA of *TL* results for Malta Plateau environment,using direct evaluation of *TMV*

The probability of detection P_d of a sonar system depends on two parameters – the *signal-to-noise* ratio (*SNR*) at the input of the detection system and the value of a *decision threshold*, say T_d ; if the system output is greater than or equal to T_d it is decided that the target is present, and absent otherwise.

The *SNR* (in decibels) can be computed by

$$SNR = SL + DI - 2 \cdot TL + TS - NL \qquad (14)$$

where *SL* is the source level, *DI* is the receiver directivity index, *TL* is the transmission loss*, TS* is the target strength and *NL* is the noise level. All these quantities are specified in decibels, either as ratios or relative to reference values.

In operating a sonar, T_d is commonly set to give a specified *probability of false alarm* P_{fa} . Expressions relating P_d to P_{fa} and *SNR* have been derived in numerous textbooks – see, for example Burdic (1991). For given values of *SL*, *DI*, *TS*, *NL* and P_{fa} , P_d is a monotonic function of *TL*, that is, P_d decreases as *TL* increases. Denote this monotonic function by

$$P_d = F_{Pd}(TL). \qquad (15)$$

To perform UA and SA on the output parameter P_d , we carry out the same steps as for *TL*, except that in the computation of the output ensemble, we first compute each *TL*, then apply (15).

UA and SA for probability of detection were carried out for the same source and receiver positions, and the same environment as for *TL* above. Typical values of *SL*, *DI*, *TS*, *NL* and P_{fa} were used in (14).

The results are shown in Figure 6. The solid curve in sub-plot (a) shows mean P_d versus range. The upper dashed curve shows P_d + one standard deviation (s.d.), with any values greater than 1 (due to the skewness of the distribution of P_d) clipped to 1. The lower dashed curve shows P_d - one s.d., with negative values (due to P_d skewness) clipped to zero.

The solid and dashed curves in sub-plot (b) show the relative *TMV*s of P_d due to uncertainty in *SSP* and *BL* respectively. The *TMVs* were computed using the direct evaluation method. Note that P_d is more sensitive to *SSP* than *BL* at all ranges greater than 3200m. Recall that from Figure 5(b), *TL* is more sensitive to *BL* at some ranges greater than 3200m, viz. 5500m to 7300m and 9400m to 10000m. In other words, the order of sensitivities was reversed at these ranges.

The following explanation of the insensitivity of P_d to *BL* at longer ranges is offered by Adrian Jones (private communication, 2009). The observation that the P_d at longer ranges is sensitive to the sound speed profile, and not sensitive to the seafloor, is due to a combination of the fact that relevant P_d values at these ranges are low, and the fact that the *SSP* variability scales from mostly downward refracting to a few cases of upward refraction, or near to it. This probably means that the only detections one gets at longer ranges are due to the near-upward refracting scenarios for which the seafloor is not a major factor. That is, it is the *SSP* variation that gives rise to the detections.

If the ensemble of *SSP* were *all* downward refracting, one would expect P_d to be more strongly influenced by the seafloor, if the seafloor varied from absorptive to reflective.

So one would expect that the relative sensitivities of P_d to uncertainty in SSP and uncertainty in seafloor reflectivity are a strong function of (i) the nature of the "baseline" (unperturbed) *SSP* and of the assumed distribution for the *SSP*s due to uncertainty, and (ii) the nature of the "baseline" seafloor reflectivity and the assumed distribution for this due to the uncertainty. Another factor is likely to be whether the Pd is either (1) all high, (2) all low, (3) in between. If P_d is either all high or all low, the sensitivity to parameter uncertainty will be due to the conditions which give rise to the outliers.

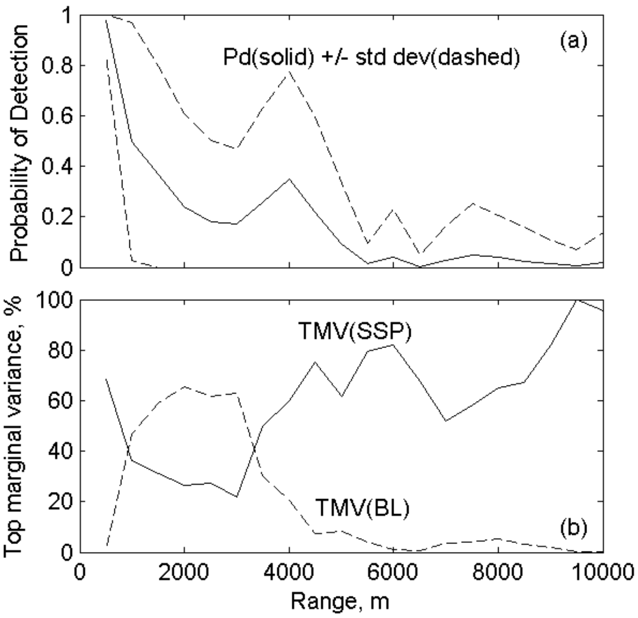


Figure 6. (a) UA and (b) SA of P_d results for Malta Plateau environment,using direct evaluation of *TMV*

CONCLUSIONS AND FUTURE WORK

This paper presents two enhancements to an underwater acoustic sensitivity tool (UAST v.1). These are (i) direct evaluation of sensitivity, as measured by top marginal variance, which is more accurate than the resampling method, but slower; and (ii) evaluation of uncertainty and sensitivity in probability of detection (P_d), an important measure of sonar performance.

The uncertainty and sensitivity in P_d versus range were compared with those of transmission loss (*TL*) versus range for the Malta Plateau environment. It was found that at some ranges (including ranges greater than 3200m), *TL* was more sensitive to bottom loss uncertainty than sound speed profile (*SSP*) uncertainty; however, for the parameters used, P_d was more sensitive to *SSP* uncertainty at all ranges greater than 3200m. An explanation of these results due to Jones (2009) is given.

In future work, the UAST will be enhanced to allow (i) more than two uncertain parameters; (ii) Latin hypercube sampling, which covers the sample hyperspace more thoroughly, and allows a reduction in the number of samples, (iii) modelling of *SSP* uncertainty by a set of independently varying orthogonal functions (iv) modelling of uncertainties in several bottom parameters, (v) using other acoustic models for *TL* and (vi) modelling of uncertainties in other parameters in P_d , such as target strength, noise level uncertainty, positional uncertainty, etc. The tool will be used on a large number of scenarios to glean a better understanding of the relative importance of uncertainties in inputs in various environments.

REFERENCES

Burdic, W.S. 1991, *UnderwaterAcoustic Systems Analysis (2nd edition)*, Prentice-Hall, Englewood Cliffs, NJ

Dosso, S.E., Giles, P.M., Brooke, G.H., McCammon, D.F., Pecknold, S & Hines P.C., 2007, 'Linear and nonlinear methods of ocean acoustic environmental sensitivity', *J. Acoust. Soc. Am.* Vol.21, no.1, pp.42-45.

Jansen, M.J.W. 2005, *ASSA: Algorithms for stochastic sensitivity analysis,* Working Document 4, Wageningen University and Research Centre, The Netherlands.

Jones, A.D., Day, G.J. and Clarke, P.A. 2008, 'Single parameter description of seafloor for shallow oceans', *Proceedings of Acoustics '08 Conference,* 29 June – 4 July, Paris, France, pp.1275-1730.

McKay, M.D., Beckman, R.J. and Conover, W.J., 1979, 'A comparisom of three methods for selecting values of input variables in the analysis of outputs from a computer code', *Technometrics* Vol.21, pp.239-245.

Parratt, L.G., 1961, *Probability and Experimental Errors in Science*, John Wiley and Sons, New York.

Porter, M.B. 2008, *Acoustics Toolbox (AcT)* http://www.hlresearch.com/ oalib/Modes/AcousticsToolbox/

Saltelli, A., Chan, K. & Scott, E.M. (eds.) 2000, *Sensitivity Analysis*, John Wiley and Sons.

Saltelli, A., Tarantola, S., Campolongo, F. & Ratto, M. 2004, *Sensitivity Analysis in Practice.* John Wiley and Sons.

Simakov, S., 2009, 'On preservation of the relation of order between top marginal variances under a mapping defined by a monotonic function', Notes prepared for D.R.Sweet.

Sweet, D.R. and Jones, A.D., 2008, 'A tool to estimate the sensitivity of underwater acoustic transmission loss to environmental uncertainty', *Proceedings of Acoustics 2008*, 24-26 November 2008, Geelong, Australia, Paper No. 71.