Experimental observation of the frequency dependence of horizontal refraction effect on bearing estimation of hydroacoustic events

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ABSTRACT

The large-scale inhomogeneity of the ocean environment and bathymetry potentially leads to horizontal refraction of underwater sound propagation and consequently induces errors in estimation of bearing to hydroacoustic events located by a receive array. The mode and frequency dependent horizontal refraction in the ocean region between the Sumatra fault zone and the Comprehensive Nuclear-Test-Ban Treaty (CTBT) hydroacoustic station deployed off Cape Leeuwin in Western Australia (HA01 station) is numerically investigated in this paper. Errors of bearing from the HA01 station to low-frequency seismic events in the Sumatra coastal zone due to horizontal refraction are also calculated for different modes and frequencies. The back-azimuth estimation for the location of the Great Sumatra-Andaman Earthquake from HA01 was conducted by analysing the received signals in three different frequency bands with the central frequencies of 4.5 Hz, 7.5 Hz and 10.5 Hz. It is found that the back-azimuth estimates change with frequency. The difference between estimates at 4.5Hz and 7.5 Hz is nearly half a degree, while that between 7.5 Hz and 10.5Hz is noticeably smaller. These observations agree with the numerically predicted results, confirming the frequency dependence of horizontal refraction and demonstrating its effect on bearing estimation of hydroacoustic events.

INTRODUCTION

A network of hydroacoustic stations was deployed worldwide, as part of the International Monitoring System (IMS) of the Comprehensive Nuclear-Test-Ban Treaty, to monitor the oceans for underwater explosions [http://www.ctbto.org/]. There are six hydrophone stations of the IMS hydroacoustic network in the Indian Ocean (off Cape Leeuwin, Western Australia; off Crozet Island; and off the Chagos Archipelago, Diego Garcia US Navy support facilities), Pacific Ocean (off Queen Charlotte Island, Canada; off Juan Fernandez Island) and Atlantic Ocean (off Guadeloupe). Each hydrophone station has either one or two so-called triplets, i.e. triangular horizontal arrays of three hydrophones separated about 2 km from each other. This design allows estimation of bearing to remote low-frequency hydroacoustic events [Chapp et al, 2005]. The hydrophones are submerged near the SOund Fixing and Ranging (SOFAR) channel axis at a depth of about 1100 meters, so that they are capable of long-range acoustic reception. Since the deployment of the hydroacoustic stations, various hydroacoustic events have been detected and located. These hydroacoustic events include in particular seismic activity [Hansen and Bowman, 2005 and 2006; Tolstoy and Bohnenstiehl, 2006] and Antarctic ice-related events [Chapp et al, 2005; Talandier et al, 2006; Li and Gavrilov, 2006 and 2008; Gavrilov and Li, 2007 and 2008].

In order to enhance the accuracy of bearing estimates for hydroacoustic monitoring, and subsequently the localization of hydroacoustic events, a bearing calibration scheme has been proposed to examine all possible errors of locating remote sources of underwater noise from the hydroacoustic stations [Li et al, 2008]. As part of the scheme, potential effects of horizontal refraction on the bearing estimation due to large-scale spatial variations of the sea depth and oceanographic characteristics [Jensen et al, 2000; Doolittle et al, 1988], have also been numerically investigated in our previous studies [Li et al, 2008 and 2009]. The numerical modelling results revealed that, in deep water, the effect of horizontal refraction can contribute as much as ±1° errors to the bearing estimation for sources of noise located in the Indian and Southern Oceans from both the HA01 and Diega Garcia South (HA08S) hydroacoustic stations, and the effect is highly dependent on azimuth and range to the noise source.

In this paper, a case study is presented to investigate the frequency dependence of the horizontal refraction effect on bearing estimation from the hydroacoustic stations. The mode and frequency dependence of the horizontal refraction effect in the ocean region between the Sumatra fault zone and the HA01 hydroacoustic station is numerically modelled in the first place, and then the bearing errors from the HA01 station to seismic events in the Sumatra coastal zone due to horizontal refraction are predicted for different modes and frequencies. The numerical results are compared with the back-azimuth estimation to the Great Sumatra-Andaman Earthquake by analysing the received signals in different frequency bands.

NUMERICAL MODELLING

Methodology

To numerically investigate the horizontal refraction effect of long-range low-frequency sound propagation, we follow the computational method proposed for the analysis of the Perth-Bermuda propagation experiment results [Heaney et al, 1991]. This method involves a combination of an adiabatic mode theory in the vertical dimension and a ray theory in the horizontal dimension. It takes into account horizontal refraction of individual modes due to both transverse sound speed gradients and bottom interaction over the continental slopes and sea mounts. The ray model was constructed on the surface of the Earth represented by an ellipsoid of rotation and expressed in terms of the parameters ϕ , λ and α , where

ϕ and λ are the latitude and longitude respectively, and α is the azimuth angle measured clockwise from the north. The ray equations on an ellipsoid are:

$$\dot{\phi} = \cos\alpha / \mu(\phi) \quad (1a)$$

$$\dot{\lambda} = \sin\alpha / \nu(\phi)\cos\phi \quad (1b)$$

$$\dot{\alpha} = \frac{\sin\alpha}{\nu(\phi)}\tan\phi - \left(\frac{\sin\alpha}{\mu(\phi)}\frac{\partial}{\partial\phi} - \frac{\cos\alpha}{\nu(\phi)\cos\phi}\frac{\partial}{\partial\lambda}\right)\log\kappa_n \quad (1c)$$

where k_n are the horizontal wavenumbers of modes and the variables μ and ν are:

$$\mu(\phi) = r_{eq}(1-\varepsilon^2)/(1-\varepsilon^2\sin^2\phi)^{3/2}$$

$$\nu(\phi) = r_{eq}(1-\varepsilon^2\sin^2\phi)^{1/2}, \quad (2)$$

where r_{eq} and ε are the equatorial radius and eccentricity of the Earth respectively. The last term in Eq. (1c) accounts for distortion of the ray paths due to gradients of the horizontal wavenumber k_n based on the Snell's law. If this term is neglected, the solutions of Eq. (1) are geodesics on the ellipsoid [Bomford, 1980, P649].

The KRAKEN program [Porter and Reiss, 1984] is used to calculate the modal wavenumbers on a horizontal grid with the grid size of 0.1°. The sound speed profiles are derived from the climatology salinity and temperature data gridded to 1-degree resolution in the World Ocean Atlas 2005 [Locarnini et al and Antonov et al, 2006] and then interpolated into a 0.1° grid. The bathymetry data were taken from the ETOPO2 Global 2-Minute Gridded Elevation Data (http://www.ngdc.noaa.gov/mgg/fliers/01mgg04.html). A 4-th or 5-th order Runge-Kutta method [Press et al, 2007] is used to solve the system of ordinary differential equations given by Eq. (1). In the integration process, the maximum integration increment in distance is limited by the grid size and the modal wavenumbers are interpolated within the current grid cell in order to reduce errors of numerical integration.

Spatial variations of modal wavenumbers

Figure 1 shows the spatial variations of mode 1 wavenumbers in the Eastern part of the Indian Ocean with the resolution of 0.1°. The season is winter and the frequency is 10.5 Hz. The sound speed profile and bathymetry along the path from the HA01 station to the approximate centre of the Sumatra fault zone (0.83N, 95.05E) indicated by the red line in Fig.1, are shown in Figure 2. As can be seen from Figure 1, the wavenumber has noticeable latitude dependence, with the wavenumber decreasing gradually from south to north. This is because the wavenumber is mainly determiend by sound speed profile rather than bathymetry in the deep water region. As shown in Fig.2, the SOFAR channel in the southern region is sharper with smaller sound speed at the axis than that in the northern region and therefore, the wavenumbers in the southern part of the region are higher. Due to the grid resolution, the wavenumber variation in the shallow water region cannot be clearly seen in this map.

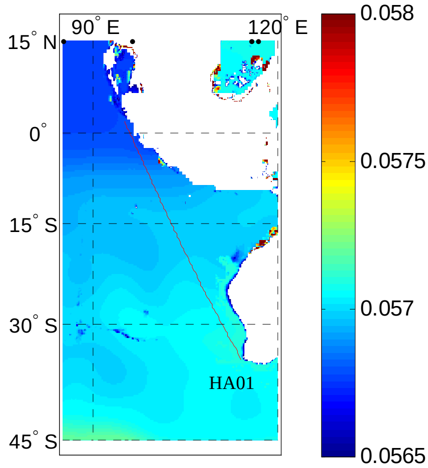

Figure 1. The spatial variations of the horizontal wavenumber of mode 1 in the eastern part of Indian Ocean. The frequency is at 10.5 Hz and climatology data are taken for the winter season. The horizontal resolution is 0.1° for both latitude and longitude. The red line connects a point chosen in the Sumatra coastal zone (0.8263N, 95.0476E) and HA01 hydroacoustic station.

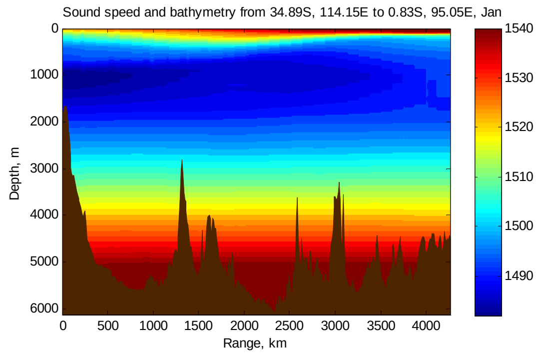

Figure 2. The sound speed profile and bathymetry along the propagation path shown by the red line in Figure 1, from the HA01 hydroacoustic station to the Sumatra coastal zone

Efect of horizontal refraction on bearing estimation

As follows from Eq. (1), the rate of azimuth deviation away from the local geodesic, governed by the last term of Eq. (1c), is proportional to logarithm of the transverse gradient of an equivalent refraction index $N_n = k_n / k_{n0}$, where the subscript zero refers to an arbitrary reference value. We selected the geodesic transect as indicated by the red line in Fig.1, to examine the influence of wavenumber gradients on the azimuth deviation. The gradients are considerably stronger in the direction along the transect, so that the rays perpendicular to this transect will experience the maximum deviation due to horizontal refraction, while the rays lying along the transect are expected to be less affected by the refraction.

The rate of azimuth deviation from the geodesic line for ray trajectories of different modes at different frequencies along the transect in the Indian Ocean is shown in Fig3. The strongest azimuth deviation rate takes place in the beginning section of the transect, where it runs over the shallow and sloping continental shelf as can be seen in Fig.2. The deviation rate due to bathymetry variations increases with the interaction between modes and the seafloor. In particular, the deviation rate increases with mode number at a given frequency, as shown in the top panel of Fig.3 and decreases with frequency for a given mode, as shown in the bottom panel of

Fig.3. The bathymetry variations across the ocean ridges along the transect also have slight impacts on the horizontal refraction of the higher-order modes, as can be seen in the top panel of Fig.3. The variation of the sound speed profile in this region induces little horizontal refraction, especially for mode 1 at higher frequencies. It is clearly seen in Fig. 3 that the effect of spatial variations of the sound speed on horizontal refraction along this path is negligible compared to that of bottom topography. Therefore, possible errors in the sound speed model based on the climatology data are also expected to be negligible.

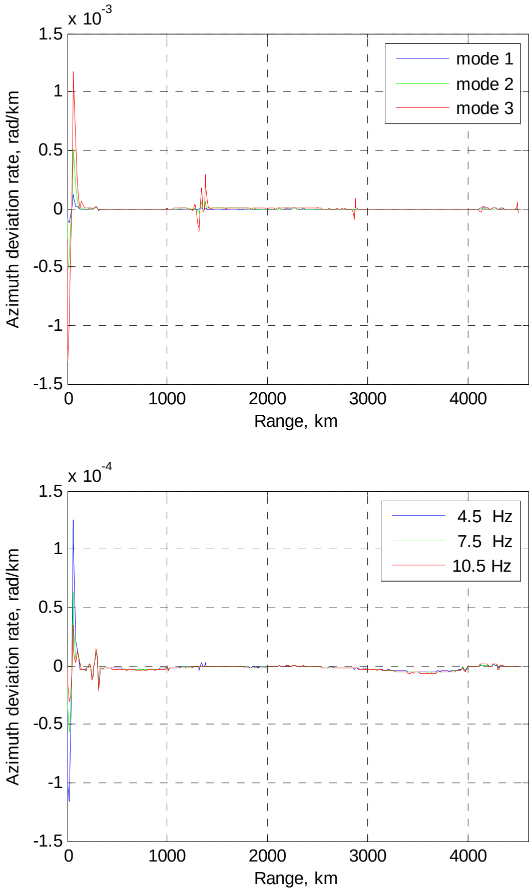

Figure 3. The azimuth deviation rate of rays from the geodetic line due to the horizontal wavenumber gradient along the transect from the HA01 station to the Sumatra fault zone. Top panel shows the deviation rate of the first three modes at 4.5 Hz and the bottom panel shows the deviation rate for mode 1 at frequencies of 4.5Hz, 7.5Hz and 10.5Hz.

Bearing errors due to horizontal refraction

From Eq. (1c), we can see that the horizontal refraction of sound propagation depends on the transverse gradient of horizontal wavenumber along the propagation path. Therefore, the deviation of bearing to hydroacoustic events due to horizontal refraction observed at the HA01 station strongly depends on the azimuth of the signal arrival at the station. The bearing deviation from the geodetic azimuth governed by Eq. (1) was calculated for each grid point as the residual $\Delta\theta = \theta_1 - \theta_2$, where θ_1 is the actual azimuth angle of the propagation path at the receiver and θ_2 is the true azimuth to the grid point as seen from the HA01 station.

The bearing deviation from the true azimuth observed from the HA01 station for noise sources located in the eastern part of the Indian Ocean is demonstrated in Fig 4. The deviations due to horizontal refraction were calculated for the spatial variations of the wavenumber of mode 1 at 4.5 Hz. The bearing deviation for HA01 does not exceed 0.2° for most parts of the region with the azimuth from HA01 less than 237°. The bearing errors are much higher for the area observed from HA01 at the back-azimuth of more than 237°, which is due to the horizontal refraction induced by the bathymetry variation over the shallower continental shelf.

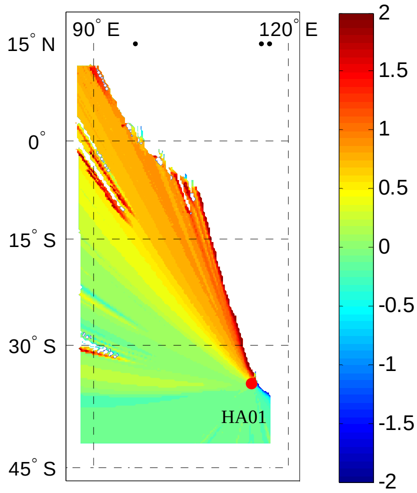

Figure 4. The map of bearing deviation in degree from the true azimuth observed from the HA01 hydroacoustic station for noise sources located in the eastern part of the Indian Ocean. The errors are due to horizontal refraction calculated for the spatial variations of the wavenumber of mode 1 at frequency of 4.5 Hz.

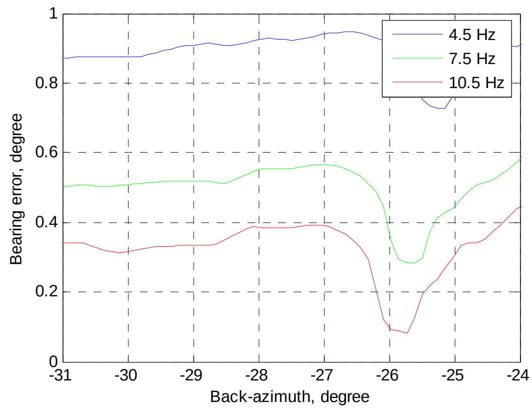

Figure 5. Errors of bearing from the HA01 station to the Sumatra fault zone due to horizontal refraction numerically predicted for mode 1 at frequencies of 4.5Hz, 7.5Hz and 10.5Hz.

Errors of bearing from the HA01 station to the Sumatra fault zone due to horizontal refraction were calculated for mode 1 at three frequencies: 4.5 Hz, 7.5 Hz and 10.5 Hz, as presented in Fig. 5. Overall, the bearings for the entire Sumatra coastal area have clockwise deviations and the deviations reveal strong frequency dependence. At 4.5 Hz, the actual signal arrival azimuth has the largest deviation from the true azimuth, which is nearly 1°. Bearings at 7.5 Hz have nearly 0.5° deviation, followed by the smallest deviations at 10.5 Hz. It is also necessary to note that the difference between bearings at 4.5Hz and 7.5 Hz is about 0.5°, while that between 7.5 Hz and 10.5 Hz is noticeably smaller.

BEARING ESTIMATION FOR THE GREAT SUMATRA-ANDAMAN EARTHQUAKE

The distance from the Sumatra coastal zone to the HA01 station is of an order of 10^3 km, which is much larger than the dimension of the triangular hydroacoustic array. Therefore, a Plane Wave Fitting (PWF) method [Del Pezzo and Giudicepietro, 2002] can be applied to estimate the back-azimuth to hydroacoustic events detected at the receive station. The arrival time differences $t_{i,j}$ for each pair of hydrophones in the triad were estimated through cross-correlation of the received signals. The relation between the arrival time difference **t** and the horizontal slowness (inverse of sound speed) **p** of plane wave propagation can be written as a vector product

$$\mathbf{t} = \Delta\mathbf{x}\cdot\mathbf{p} \qquad (3)$$

where $\Delta\mathbf{x}$ denotes the relative position of hydrophones $\{x_i, y_i\}$ and **p** is a two-element vector with horizontal components $\{p_x,p_y\}$. Because the number of linear equations in Eq.3 is larger than the number of unknown variables p_x and p_y, Eq.3 can be solved with respect to **p** in the least-mean-squares sense, which can be expressed as

$$\mathbf{p} = (\Delta\mathbf{x}^{T}\Delta\mathbf{x})^{-1}\Delta\mathbf{x}^{T}\,\mathbf{t} \qquad (4)$$

where T denotes the matrix transpose operation. The back-azimuth α and the group velocity v can then be calculated as $\alpha = \tan^{-1}(p_x/p_y)$ and $v = 1/|\mathbf{p}|$.

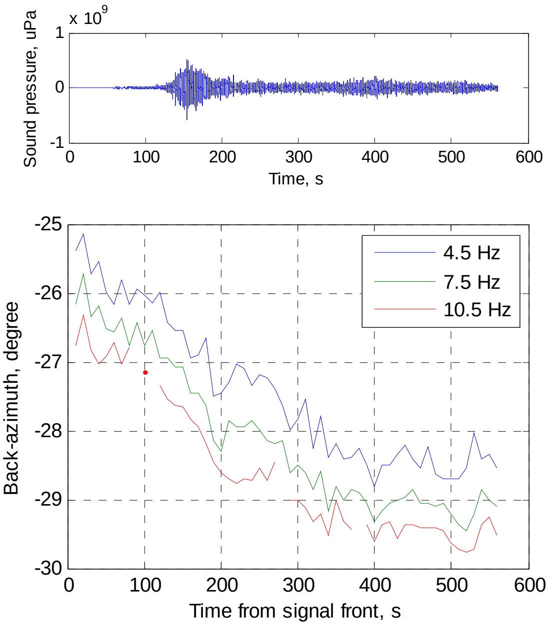

Figure 6. Top panel: An initial 10-minute fragment of the hydroacoustic signal received on one of the three hydrophones of HA01 from the mainshock of the Great Sumatra-Andaman Earthquake; Bottom panel: the back-azimuth estimates to the earthquake made from HA01 in three different 1/3-octave frequency bands with the central frequencies of 4.5Hz, 7.5Hz and 10.5Hz. The RMS error of back-azimuth estimates is about 0.2°, which was demonstrated in [Li *et al*., 2008]

As one of the largest recorded earthquakes, the Great Sumatra-Andaman earthquake occurred on 26 December 2004 [Lay et al, 2005]. An initial 10-minute fragment of the hydroacoustic signal received on on of the HA01 hydrophones from this seismic event is shown in the top panel of Fig.6. These 10-minute signals received in each channel were divided into sections of 10 seconds long and the back-azimuth estimates for each section were then made in three different 1/3-octave frequency bands with central frequencies of 4.5Hz, 7.5 Hz and 10.5Hz. The back-azimuth estimates over all of the 10-second sections are presented in the bottom panel of Fig. 6. For all three frequency bands, the variations of the back-azimuth in the first 8 minutes reveal anti-clockwise motion of the signal source of nearly 3°. This is so because the earthquake rupture propagated to the North-Northwest (NNW) along the Sumatra fault zone during this period [Ammon et al, 2005].

It can also be noted that the measured back-azimuth decreases with frequency. The difference between the back-azimuths measured at 4.5 Hz and at 7.5 Hz is slightly over 0.5°, while that between 7.5 Hz and 10.5 Hz is noticeably smaller. These observations agree with the modelled results shown in Fig.5. Compared with other possible errors of bearing from the HA01 station [Li et al, 2008], the effect of horizontal refraction is the major factor that causes the frequency dependence of bearing estimation for the Great Sumatra-Andaman event.

CONCLUSIONS

This study presents the numerical modelling results and observation of bearing to the Great Sumatra-Andaman Earthquake from the HA01 hydroacoustic station. Both modelling and observation of the bearing reveal strong frequency dependence due to horizontal refraction, which is mainly induced by bathymetry variations over the continental shelf region. This study also demonstrates that the effect of horizontal refraction on bearing estimation from the IMS hydroacoustic stations can be significant in the presence of other errors for certain locations of underwater noise events in the ocean. Possible effects of ocean currents on horizontal refraction of acoustic waves propagated along the path from Sumatra to HA01 were not considered in this study, although it would be worthwhile to estimate these effects in future studies.

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