Reduction of the sound power radiated by a submarine using passive and active vibration control

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ABSTRACT

As submarines can be detected due to their sound radiation, it is desired to minimise the radiated sound power. At low frequencies, submarine radiated noise is correlated to sound radiation from the propeller and the hull, where the hull is excited by propeller forces. In this paper, passive and active control of vibration is applied to a numerical model to reduce the low frequency sound radiation from a submarine. The performance of a control system using actuators that are tuned to the hull and propeller/shafting system resonance is investigated. In addition, an optimised resonance changer is implemented in the propeller/shafting system.

INTRODUCTION

Sound radiated from a submarine due to propeller forces represent a significant portion of the overall radiated sound. Fluctuating propeller forces are due to operation of the propeller in a spatially non-uniform wake. A significant part of the force variation is time-harmonic and can be related to the blade passing frequency (*bpf*) and its multiples. The forces excite vibration of the hull correlated to the accordion modes which are efficient sound radiators. In general, there are three approaches to reduce the sound radiated by a submarine due to propeller forces corresponding to (i) addressing the source mechanism of excitation, (ii) passive vibration control and (iii) active control. In order to address (i), submarine propellers feature sickle-shaped blades and the number of blades is higher than for conventional propellers. This gives a more even distribution of thrust and the passage of an individual blade through a steep wake velocity gradient occurs gradually. Passive vibration control can be implemented using a resonance changer (RC) in the propeller shafting system Goodwin (1960). An RC is a vibration attenuation device that detunes the fundamental resonance of the propeller/shafting system (PSS) and introduces damping by hydraulic means. The effect of the RC in the presence of propeller forces transmitted both through the shaft and via the fluid to the hull has been investigated by Merz et al. (2009b). Optimum parameters for the RC have been found by Dylejko (2008) and Merz et al. (2009a) for analytical and numerical models, respectively.

Using active control, it is desirable to use actuator forces that are as small as practicable. The cost, power, space and weight requirements of the control system should be minimised while maximising the reliability and robustness of the control system. One method of achieving small optimum control forces is to use added masses and springs that are tuned separately to each major resonance of the hull and shafting system. These masses and springs can also have a significant effect on hull vibration and sound radiation when the active control system is not operating, since they act as passive vibration absorbers. Gündel (2009) used active vibration control (AVC) to minimise vibration of an aircraft fuselage, where actuators were attached to the frames of the fuselage. The actuators featured passive elements that were tuned to the first three harmonics of *bpf*. AVC of low frequency hull vibration and radiated noise for a submarine has been presented recently by Pan et al. (2008a;b). They used a control moment to reduce the sound power radiated by the axially excited submarine hull. A reduction of the radiated sound power by up to two-thirds compared to the sound power radiated by the uncontrolled model was achieved.

This work utilises estimates of the fluctuating propeller forces at each speed and frequency as well as constraints on the size of the control force amplitudes, in order to select appropriate actuators for practical design of an active vibration control system for a submarine. The performance of an active vibration control system as well as the combination of a passive/active control system is presented for the case of axial excitation of a submarine hull due to propeller forces. The frequency range between 1 and 100 Hz was considered in order to address the first four harmonics of *bpf*. As the excitation is related to *bpf*, feedforward control has been implemented. Excitation of the hull through the propeller/shafting system and excitation of the submarine hull via the fluid have been taken into account. In addition, the sound field directly radiated from the propeller has been considered for the computation of the overall radiated sound power. The secondary excitation was applied using actuators with tuned passive elements. The passive elements of the actuators were tuned such that their natural frequencies are similar to the natural frequencies of the pressure hull or to the fundamental resonance of the propeller/shafting system, in order to reduce the required control force at peak sound radiation.

A vibro-acoustic model of a submerged submarine has been developed, where the structure is represented by finite elements and the acoustic medium is represented by boundary elements. The performance of a combined passive/active control system using a resonance changer implemented in the propeller/shafting system as the passive control mechanism and the tuned actuators is also presented.

PHYSICAL MODEL OF THE SUBMARINE

A simplified physical model of a submarine has been developed, where the pressure hull is represented by a thin-walled cylindrical shell with ring stiffeners. The pressure hull has rigid flat end plates and is subdivided into three compartments by two evenly spaced bulkheads, as shown in Fig. 1. A rigid conical shell is attached to the stern end plate to represent the tail cone. Lumped masses are attached to the end plates in order to account for the ballast tank, casing and the mass of the water entrained in the end closures. The on-board machinery and internal structure has been considered as a mass added to the cylindrical pressure hull (Tso and Jenkins 2003).

The dynamics of the propeller/shafting have been modelled for the axial direction using a modular approach (Dylejko et al. 2007), as shown in Fig.2. The propeller and mass of water entrained at the propeller have been considered as a lumped mass m_p, where the external force and the structural velocity in the axial direction at the propeller are given by f_p and v_p, respectively. The propeller shaft is represented by a simple rod of overall length l_s, where only a part of the rod having length l_{se} is fully dynamically active. The remaining part is treated as an attachment and modelled as a lumped mass. The cross sectional area, Young's modulus and density of the rod are given by A_s, E_s and ρ_s, respectively. The dynamic behaviour of the thrust bearing has been reduced to a spring-mass-damper system, where the mass, damping and stiffness coefficients are given by m_b, c_b and k_b, respectively. The resonance changer has been simplified to a spring-mass-damper system according to Goodwin (1960), with virtual stiffness k_r, mass m_r and damping c_r parameters. The foundation is represented by an axisymmetric thin-walled conical shell, where the minor and major radii are given by a_f and b_f, respectively. The remaining properties of the foundation are given by the Young's modulus E_f, density ρ_f, Poisson's ratio ν_f and thickness h_f. The foundation is assumed to be welded to the stern end plate at its major radius, where the axial force acting on the hull and the axial velocity of the end plate are given by f_h and v_h, respectively.

Propeller Model

As the fluid wave length is large compared to the propeller diameter, the sound radiation from the propeller has been considered as a superposition of two axial acoustic dipoles. The first dipole is due to the hydrodynamic mechanism that arises from the propeller operating in a non-uniform wake and the second dipole is due to axial fluctuations of the propeller (Merz et al. 2009b). The strength of the dipole due to the hydrodynamic mechanism is directly associated with the structural force applied to the propeller hub and the strength of the dipole due to propeller vibration is evaluated by using the radiation impedance of a fluctuating rigid disc (Mellow and Kärkkäinen 2005). Under the assumption that the propeller can be simplified to a rigid disc, the pressure field is given by

$$p(r,\theta) = \mathrm{j}k\mathrm{g}(r)\left(1 - \frac{\mathrm{j}}{kr}\right)(f_p + 2sz_c z_a v_p)\cos\theta, \tag{1}$$

where k is the fluid wave number, θ is the angle between the field point direction and the force direction, f_p is the amplitude of the exciting force at the propeller hub due to the hydrodynamic mechanism, r is the distance between the source and the field point, $s = \pi a^2$ is the area of the disc surface, z_c is the characteristic impedance of the fluid and z_a is the radiation impedance.

$$\mathrm{g}(r) = \frac{\mathrm{e}^{-\mathrm{j}kr}}{4\pi r} \tag{2}$$

is the free space Green's function. The radiation impedance can be expressed as the sum of its real and imaginary parts, corresponding to the resistance r_a and the reactance x_a, respectively. The resistance and reactance can be obtained under the assumption that a freely suspended disc has twice the admittance of a disc in an infinite baffle (Mellow and Kärkkäinen 2005). For small ka, this gives

$$r_a = \frac{8(ka)^4}{27\pi^2}, \quad x_a = \frac{4ka}{3\pi}. \tag{3}$$

Active Control System

In order to efficiently address peak sound radiation at the axial resonances of the hull and propeller/shafting system, actuators with additional passive elements are used (Gündel 2009). They feature a mass m_a that is linked to a base by a spring of stiffness k_a and a damper of damping coefficient c_a, as shown in Fig. 2. A pair of control forces f_a of opposite direction to the mass and base is introduced by an electromagnet. The base is fixed to the stern end plate in order to address vibration correlated to the hull resonances, or to the thrust collar to address vibration correlated to the fundamental resonance of the propeller/shafting system. In order to take into account the axisymmetric nature of the problem, a set of actuators that are tuned to a certain frequency are evenly distributed on the end plate over a virtual concentric ring around the propeller shaft. Three sets of actuators are attached to the end plate, where each set of actuators is tuned to respectively control the first, second and third axial resonances correlated to the accordion modes of the submarine hull. The actuator used to address the fundamental resonance of the propeller/shafting system is attached to the thrust collar. This could be accomplished by using a freely suspended mass that interacts with the thrust collar through an electromagnet only, where the magnet can simultaneously act as spring, damper and force (Lewis and Allaire 1989).

FE/BE MODEL

The fitness of the active control systems is investigated using a numerical simulation in the frequency domain, where the structure is represented by finite elements and the acoustic domain is represented by boundary elements. The overall radiated sound power or the axial velocity of the stern end plate are used as cost functions to be minimised. For the pressure hull, ring-stiffeners, bulkheads and the foundation of the propeller/shafting system, axisymmetric Reissner-Mindlin shell elements with quadratic interpolation functions have been used (Ahmad et al. 1970). Simple linear rod elements are used for the propeller shaft (Zienkiewicz and Taylor 2005). The remaining dynamic components corresponding to the propeller, the ineffective part of the propeller shaft, the thrust bearing, the RC and the lumped masses at the end plates are represented by spring-damper and mass elements. The dynamics of the uncoupled structure can be represented by the structural stiffness matrix $\mathbf{K}$, the frequency dependent damping matrix $\mathbf{C}_f$, the constant damping matrix $\mathbf{C}_c$ and the mass matrix $\mathbf{M}$.

For the acoustic domain, the direct boundary element method has been used in order to solve the Kirchhoff-Helmholtz integral equation (Seybert et al. 1986). In order to discretise the integral equation, the point collocation method was applied. This results in the relation

$$\mathbf{G}_\Gamma \mathbf{v}_n + \mathbf{p}_{inc} = \mathbf{H}_\Gamma \mathbf{p}_\Gamma, \tag{4}$$

where $\mathbf{v}_n$ and $\mathbf{p}_\Gamma$ are the normal velocities and the acoustic surface pressures at the collocation points on the radiating surface Γ. $\mathbf{G}_\Gamma$ and $\mathbf{H}_\Gamma$ are called BEM influence matrices. The incident pressure $\mathbf{p}_{inc}$ at the collocation points is due to fixed acoustic sources such as dipoles.

As non-conforming meshes have been used for the finite element (FE) model and the boundary element (BE) model, the mortar element method was applied in order to couple the structural domain to the acoustic domain (Belgacem 1999). This requires that the continuity condition between the fluid and the structural surface is relaxed and piecewise sustained in a weak sense. The coupled FE/BE system can formally be written as

$$\begin{bmatrix} \mathbf{K} + j(\omega\mathbf{C}_f + \mathbf{C}_c) - \omega^2\mathbf{M} & \mathbf{R}_{sf} \\ \mathbf{G}_\Gamma \mathbf{R}_{fs} & \mathbf{H}_\Gamma \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p}_\Gamma \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_s \\ \mathbf{p}_{inc,\Gamma} \end{Bmatrix}, \tag{5}$$

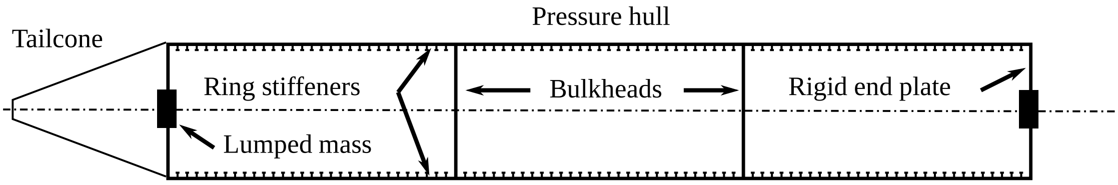

Figure 1: Submarine model

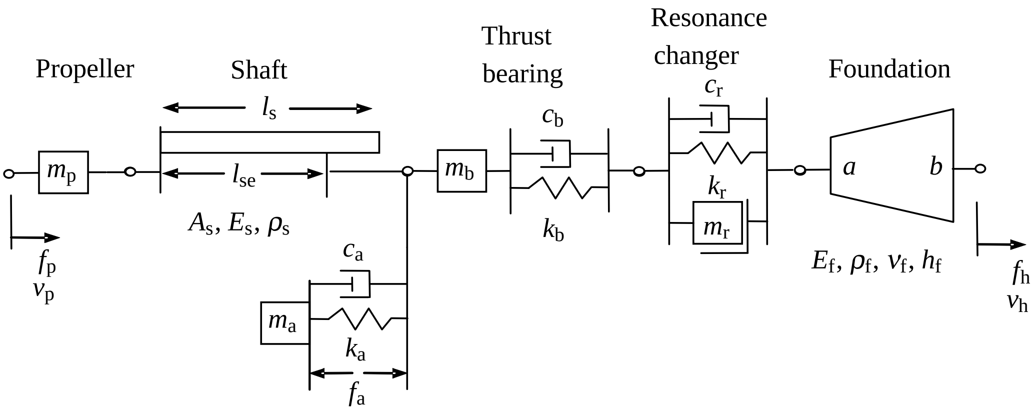

Figure 2: Modular approach for the propeller/shafting system with an active control system

where $\mathbf{R}_{\mathrm{fs}}$ and $\mathbf{R}_{\mathrm{sf}}$ are the mortar coupling matrices and $\mathbf{f}_{\mathrm{s}}$ is the vector of external structural point forces. The overall radiated sound power can be computed by

$$\Pi = \mathbf{p}_{\Lambda}^{\mathrm{H}} \mathbf{\Theta} \mathbf{p}_{\Lambda}, \tag{6}$$

where $\mathbf{p}_{\Lambda}$ is the pressure at a set of sensibly chosen integration points on a spherical surface Λ that surrounds the structure and any acoustic sources. The diagonal matrix $\mathbf{\Theta}$ is established using numerical integration and fluid property data. The acoustic field pressure $\mathbf{p}_{\Lambda}$ can also be computed using the Kirchhoff-Helmholtz integral equation

$$\mathbf{p}_{\Lambda} = \mathbf{T} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p}_{\Gamma} \end{Bmatrix} + \mathbf{p}_{\mathrm{inc},\Lambda}, \tag{7}$$

where $\mathbf{p}_{\mathrm{inc},\Lambda}$ is the pressure contribution at the integration points from acoustic sources. The transfer matrix $\mathbf{T}$ is composed of the BE transfer matrices $\mathbf{G}_{\Lambda}$ and $\mathbf{H}_{\Lambda}$ and the mortar coupling matrix $\mathbf{R}_{\mathrm{fs}}$ and is given by

$$\mathbf{T} = \begin{bmatrix} \mathbf{G}_{\Lambda}\mathbf{R}_{\mathrm{fs}} & \mathbf{H}_{\Lambda} \end{bmatrix}. \tag{8}$$

The FE matrices were obtained using ANSYS 11. The BE and coupling matrices have been computed using software written by the authors that has been implemented using SciPy and C++. At least 10 elements per wave length were used for the structural and acoustic meshes. As the fluid wave length is larger than the structural wave length, non-matching meshes were employed.

ACTIVE CONTROL MODEL

Since the excitation of the vibro-acoustic system is deterministic, feedforward control is used. The multi-channel vibro-acoustic system depicted in Fig. 3 is considered, where $\mathbf{f}_{\mathrm{p}}$ and $\mathbf{f}_{\mathrm{s}}$ are vectors of the primary and secondary excitation spectra, respectively. $\mathbf{G}_{\mathrm{p}}$ and $\mathbf{G}_{\mathrm{s}}$ are matrices of the primary and secondary path spectra, respectively. $\mathbf{d}$ is the vector of disturbance spectra and $\mathbf{e}$ is the vector of spectra from the error sensors. As the spectra $\mathbf{f}_{\mathrm{p}}$, $\mathbf{G}_{\mathrm{p}}$ and $\mathbf{G}_{\mathrm{s}}$ are known, it is possible to find the optimum spectra for the secondary excitation $\mathbf{f}_{\mathrm{s}}$ in order

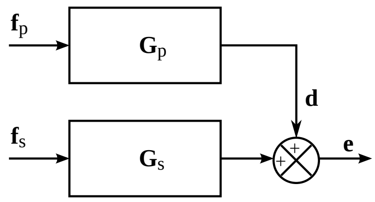

Figure 3: Multi-channel control system

to minimise a cost function obtained from $\mathbf{e}$. A cost function similar to that proposed by Fuller et al. (1996) is given by

$$J = \mathbf{e}^{\mathrm{H}}\mathbf{Q}\mathbf{e} + \mathbf{f}_{\mathrm{s}}^{\mathrm{H}}\sigma\mathbf{I}\mathbf{f}_{\mathrm{s}}, \tag{9}$$

where $\mathbf{I}$ is the unity matrix, $\mathbf{Q}$ is a hermitian, not necessarily diagonal, matrix that determines the weighting of the individual error sensor signals and $\sigma \geq 0$. As the force of the actuators have a physical limit, σ has to be found iteratively such that this limit is not exceeded (Gündel 2009). From Fig. 3 it can be seen that

$$\mathbf{e} = \mathbf{G}_{\mathrm{p}}\mathbf{f}_{\mathrm{p}} + \mathbf{G}_{\mathrm{s}}\mathbf{f}_{\mathrm{s}}. \tag{10}$$

Using equation (10) and

$$\frac{\partial J}{\partial \mathbf{f}_{\mathrm{s}}} = \mathbf{0} \tag{11}$$

it can be shown that the optimum secondary forces $\mathbf{f}_{\mathrm{s,opt}}$ are given by

$$\mathbf{f}_{\mathrm{s,opt}} = -\left[\mathbf{G}_{\mathrm{s}}^{\mathrm{H}}\mathbf{Q}\mathbf{G}_{\mathrm{s}} + \sigma\mathbf{I}\right]^{-1}\mathbf{G}_{\mathrm{s}}^{\mathrm{H}}\mathbf{Q}\mathbf{G}_{\mathrm{p}}\mathbf{f}_{\mathrm{p}} \tag{12}$$

MODEL PARAMETERS

A cylindrical pressure hull of 45 m length and 3.25 m radius has been considered which corresponds to a medium size submarine hull. The hull was modelled with evenly spaced interior ring-stiffeners of a rectangular cross-section of 0.012 m^2

and a spacing of 0.5 m. Two evenly spaced bulkheads were included. Both the bulkheads and the pressure hull have a thickness of 0.04 m. The end plates of the pressure hull and the tail cone have been considered as rigid structures. An added mass was added to the cylindrical shell in order to account for the on-board machinery, where a mass per unit area of $796\,\mathrm{kg/m^2}$ guaranteed neutral buoyancy. In addition, a lumped mass of 200 tonnes was added to the bow side end plate and another lumped mass of 188 tonnes was added to the stern end plate.

The propeller was assumed to have a diameter of 3.25 m and a structural mass of 10 tonnes. The added mass of water of the propeller has been determined as 11.443 tonnes. For the shaft, an effective length of 9 m and an overall length of 10.5 m was considered. The shaft has a cross-sectional area of $0.071\,\mathrm{m^2}$. The thrust bearing was modelled as a spring-mass-damper system with stiffness $2 \times 10^{10}\,\mathrm{N/m}$, mass 0.2 tonnes and damping $3 \times 10^{5}\,\mathrm{kg/s}$. The foundation is represented by an axisymmetric, truncated conical shell with minor radius 0.52 m, major radius 1.25 m, half angle 15° and thickness 10 mm. For all parts of the model, steel was considered with a structural density of $7800\,\mathrm{kg/m^3}$ and Poisson's ratio of 0.3. Structural components of the pressure hull and the propeller/shafting system have a Young's modulus of 210 GPa and 200 GPa, respectively.

In the numerical model, an allowance of $m_a = 1$ tonne has been included for the mass of each tuned actuator associated with the hull axial resonances and the propeller/shafting system resonance. This is sufficient to have a significant effect on these principal resonances with a damping factor of $\zeta_a = 0.02$ (loss factor = 0.04) when the control system is turned off, owing to the passive vibration absorber effect. If this benefit is not needed, then the added mass can be reduced to much smaller values. In practice, it is possible to obtain an actuator force of 1 kN rms using commercially available electromagnet shakers with a total mass of 100 kg and a total moving mass of only 2 kg; the larger mass of the magnets themselves can be used to form part of the added mass. The damping parameter for the actuators is given by $c_a = 2\zeta_a\sqrt{k_a m_a}$, where the stiffness coefficients k_a were chosen such that the natural frequency of a tuned actuator equals the corresponding hull resonance or the propeller/shafting system resonance.

RESULTS

The active control excitation system is shown in Fig. 4. A set of three tuned actuators at the stern end plate and one tuned actuator at the propeller shaft is used. In addition, the performance of combined active and passive control is investigated, where a resonance changer is implemented.

In order to demonstrate the potential of the active control system with small actuator forces, the force magnitude is initially restricted to 10% of the fluctuating propeller force at each frequency. This is a severe restriction, because electrodynamic shakers, for example, can give almost constant force amplitude

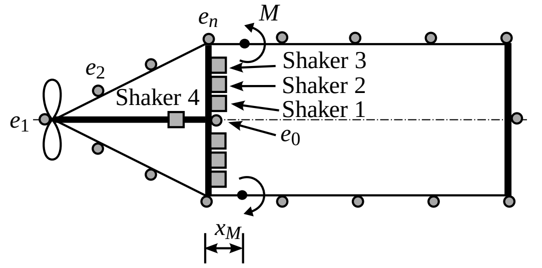

Figure 4: Control system components

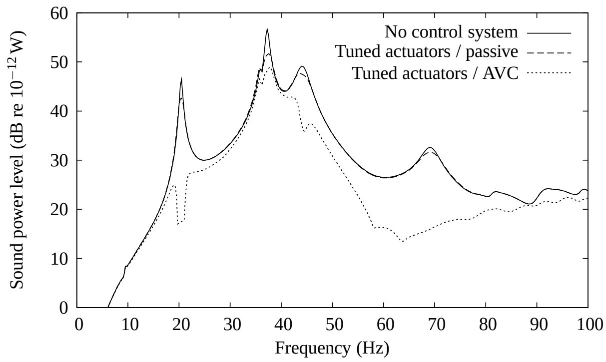

Figure 5: Performance of the control system

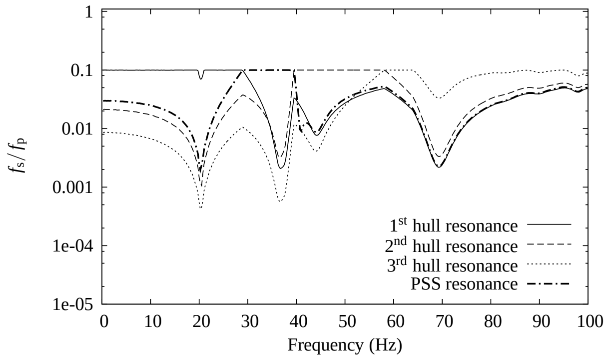

Figure 6: Control forces

over a wide frequency range from less than a few Hz to a kHz or more, while the required forces reduce with speed and also with the multiple of *bpf*. It is also demonstrated how further improvements in performance can be achieved by allowing the actuator force to be increased to 30% and then 100% of the propeller force at each frequency. AVC has been used to obtain a cost function based on axial vibration of the stern end plate. A single sensor to measure the velocity e_0 of the stern end plate was used. All frequency response functions are given for unity force excitation of the propeller and the correlating dipole excitation.

Results are compared where the control force is limited to 10% of the primary force. Figure 5 shows the total radiated sound power levels due to hull vibration and propeller sound when (i) no control system is implemented, (ii) the tuned actuators are implemented but no control force is applied, (iii) the tuned actuators are implemented and active control is used. The peaks at around 20, 44 and 69 Hz represent the first three axial resonances of the hull and the peak at around 37 Hz represents the fundamental resonance of the propeller/shafting system. It can be seen that the control system reduces the peak sound radiation at the aforementioned resonances in passive mode by 2–5 dB, for example, if the active system fails. The tuned actuators work particularly well in active mode at the hull resonances, where a reduction of the radiated sound power by up to 25 dB is achieved. The performance at the propeller/shafting system resonance is less significant.

Figure 6 presents the corresponding control forces for the tuned actuators. It can be seen that for the first and third axial hull resonances, less than 10% of the primary force is necessary for optimum control, whereas for the second hull resonance and the propeller/shafting system resonance, the system needs to be driven to its limit.

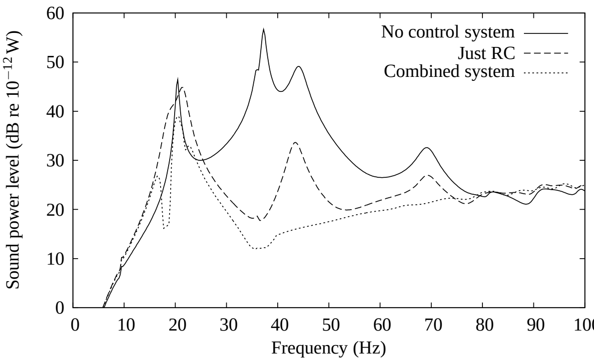

Figure 7: Performance of the control system using an RC

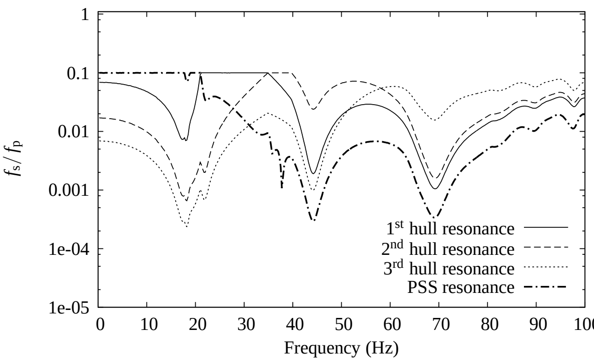

Figure 8: Control forces when an RC is present

The influence of a resonance changer (RC) on the performance of the control system is shown in Fig. 7, where optimum parameters for the RC given in (Merz et al. 2009a) corresponding to $m_r = 1$ tonne, $k_r = 5.382 \times 10^{10}$ N/m and $c_r = 1.1 \times 10^6$ kg/s were used. A significant improvement is achieved for frequencies between 25 and 60 Hz. Comparison of the results in Fig. 5 and Fig. 7 shows that the effect on sound radiation at the fundamental resonance of the propeller/shafting system is reduced considerably, when the RC is introduced. This can be attributed to the detuning of the propeller/shafting system resonance by the RC from 38 Hz to 18 Hz, which is very close to the first hull resonance at about 21 Hz. A comparison between Figs. 6 and 8 shows that less control force is required for optimum control when implementing a resonance changer for additional passive vibration control of the propeller/shafting system.

Influence of the force limit on the performance of the control system

It has been shown that a control force that is limited to 10% of the primary exciting force can significantly reduce the radiated sound power, especially for the frequency range above the first axial hull resonance. Further improvements can be achieved if the force limit is raised to 30% and 100% of the primary force. Figure 9 shows the performance of the control system using tuned actuators. An increase of the force limit to 30% leads to an improvement in performance near the hull and the propeller/shafting system resonances which is the effect of the actuators being tuned to the resonances. Figure 10 shows that an increase of the control force to 100% of the primary force enables optimum control for frequencies above 14 Hz. The dip in radiated sound power at about 53 Hz is caused by cancellation of the dipole due to the hydrodynamic mechanism by the dipole due to propeller vibration.

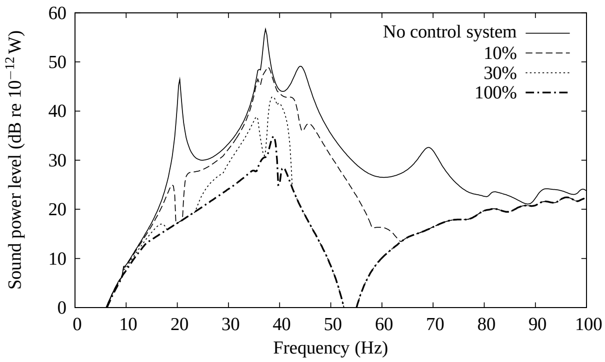

Figure 9: Performance of the control system for an increased force limit without an RC

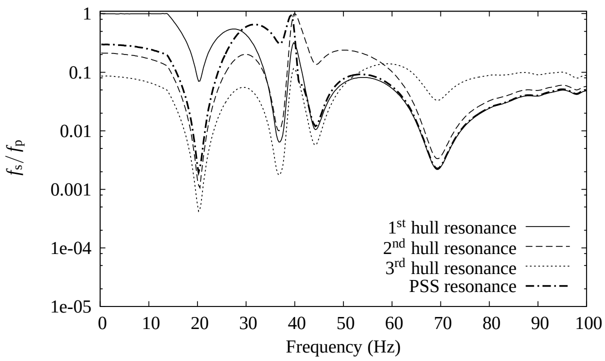

Figure 10: Control forces for 100% force limit without an RC

Figure 11 shows that a crucial reduction of radiated sound power over the whole frequency range of interest can be achieved for the control system using tuned actuators and an RC, when the control force limit is raised to 30% of the primary exciting force. A further increase of the control force limit to 100% of the primary exciting force only shows slight improvements at frequencies near the first axial hull resonance.

CONCLUSIONS

A numerical model using finite and boundary elements has been developed in order to simulate control systems implemented in a submarine that is excited by propeller forces. A control system has been implemented that employs tuned actuators at the thrust collar and the stern end plate of the hull. The performance of the stand-alone active control system and

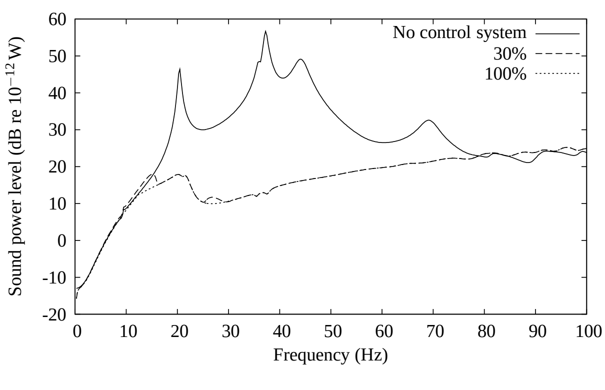

Figure 11: Performance of the system using tuned actuators, when an RC is implemented

a combination of a passive/active control system using a resonance changer have been investigated, where a limitation of the control force has been considered. It has been shown that an improvement is achieved when a passive control system is used in conjunction with the active control system, by implementing a RC. In this case, a control force that is limited to 30% of the primary exciting force guarantees optimum control for 90% of the investigated frequency range.

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