Acoustical properties of ancient Chinese musical bells

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ABSTRACT

Ancient Chinese music bells can be traced back to the Shang dynasty (1600–1100 B.C.). In addition to their significance in history and metallurgy, they provide much insight into the design of musical instruments in the early years and continue to make important contribution to acoustics due to their unique acoustical properties and rich physical mechanisms. These music bells differ from carillon/church bells and oriental temple bells by their almond-shaped cross sections, which result in two distinct strike tones (normal and side) in one bell. Through the analyse of the resonance frequencies of 64 music bells cast 2400 years ago, frequency relations are rediscovered between the normal and side-strike tones in individual bells and between the normal-strike tones of adjacent bells. Acoustical qualities of each strike tone are characterised using frequency ratios between the partials and fundamental, as well as the spectrogram of the tone. The study on the scaling rules for achieving the required frequency intervals between adjacent bells also sheds light on our understanding of music scales and sound balance principles in bell acoustics already recognized in ancient times.

INTRODUCTION

Previous studies in the acoustics of ancient Chinese music bells were mainly focused on the two-tone [1] and short-decay properties [2] of individual bells and their dependence on the nature of the bell's modal vibration and sound radiation. In practice, a set of music bells were performed together in groups as a single music instrument. It is thus necessary to investigate the acoustical properties of the entire set of bells and to identify the roles played by individual bells in determining the music scales, sound quality and balance of the instrument.

Information available for this study are the dimensions and the measured fundamentals and partials [3] of both the normal and side-strike tones of each of the 64 Marquis Zeng Yi's musical bells (see Figure 1), which were cast from more than 2400 years ago. The measured time signals of vibration and sound of the normal and side-strike tones of other 64 music bells (see Figure 2) at the Beijing bell museum are used to generate the spectrogram for tonal quality analysis. The later set of bells was made in 1990 using identical dimensions and material compositions (Cu 83%, Sn 14%, Pb 2~4%) of the former set. Signal processing techniques, boundary and finite element analysis are employed as tools in this study.

Figure 1. Marquis Zeng Yi's musical bells.

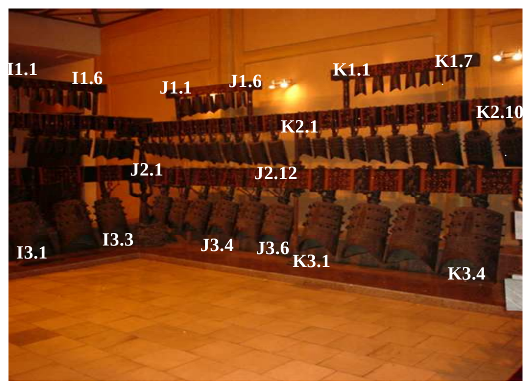


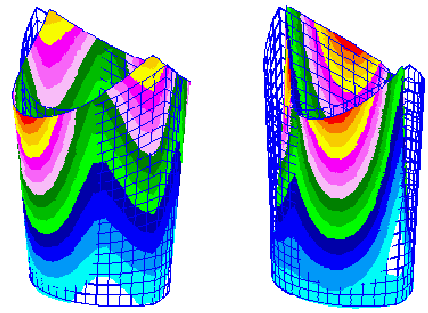
Figure 2. Sixty four musical bells at the Beijing bell museum.

The analysis of the experimental data has led to several interesting rediscoveries of the music bells, including tonal quality of the strike tones and frequency intervals between the tones. These frequency intervals demonstrate the existence of 12 semi-tones in ancient Chinese bells. Two tuning rules for achieving the required frequency intervals are also discovered together with the mechanism in implementing the rules and the significance of these rules in producing a balanced sound from these bells.

STRIKE NOTES AND THEIR SOUND QUALITY

Two tone property

Figure 3 shows the sound pressure spectra of the normal and side-strike tones of the bell K3.1 (shown in Figure 2) at Beijing bell museum. When the bell is excited near the normal-strike position (see upper Figure 3), a fundamental frequency of 110.6 Hz (A_2) is observed in the spectrum. A lower peak at 130.2Hz (C_3) is due to the contribution of the second bell mode, which can be fully excited at the side-strike position. After the initial strike, the fundamental sound component dominates. Although higher frequency partials are also excited, they decay at much faster rates than the fundamental. Consequently, a clear note of A_2 is heard. When the bell is excited near the side-strike position (see lower Figure 3), the sound pressure level at the second peak is about 20 dB higher than that at the first and a clear note of C_3 is resulted. When the bell's cross section changes from circular to almond-shaped, it lifts the degeneracy of the fundamental modes (the circumferential modes), resulting in the two-tone feature of the bells. Figure 4 shows the vibration mode shapes of the symmetrical $(2,0)_S$ (with respect to the major axis of the cross section) and anti-symmetrical mode $(2,0)_A$.



$(2,0)_S$ $(2,0)_A$

Figure 4. Symmetrical and anti-symmetrical modes of the music bells

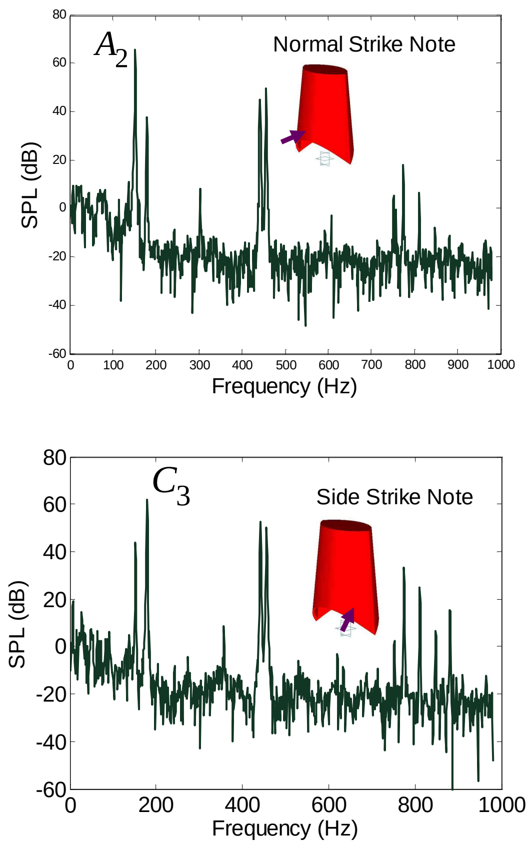


Figure 3. Frequency spectra of the normal and side-strike tones of bell K3.1 at the Beijing bell museum. Normal-strike note is at 110.6 Hz (A_2) and side-strike note is at 130.2Hz (C_3).

Tonal quality

Comparing the bells with circular cross sections, the tones of music bells of almond-shaped cross sections appear to be purer. As illustrated by the spectrograms of the two tones of bell K1.3 in Figure 5, the frequency component that determines the pitch of each tone dominates the level of the bell sound in the whole duration of sound radiation. Therefore, there is no confusion in the subjective determination of the pitch of the tones. For comparison, the spectrogram of a circular bell (tenor, $C^{\#}$ and 370kg) in the Perth swan bell museum is shown in Figure 6. Immediately after striking the bell, the fifth partial (nominal) is the most prominent partial. After the initial striking sound, the dominant role of the radiated sound was played by the second (fundamental) and third (tierce) partials and eventually by the fundamental alone. Therefore a shift in pitch of the bell sound from the higher nominal frequency to lower fundamental frequency of the circular bell is audible.

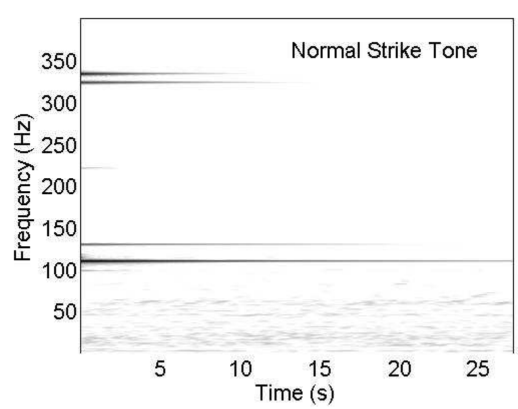


Figure 5(a). Spectrogram of the normal-strike note A_2

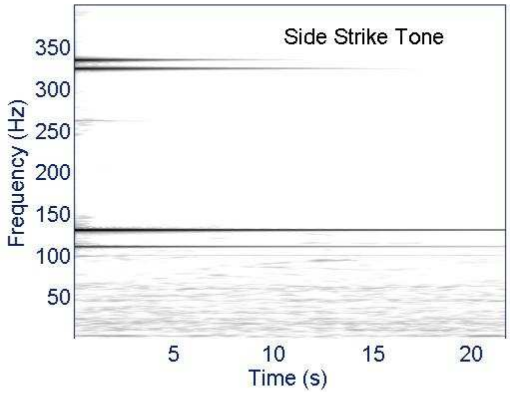


Figure 5(b). Spectrogram of the side-strike note C_3

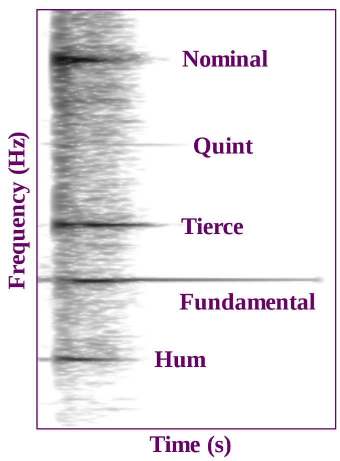


Figure 6. Spectrogram of a circular bell ($C^{\#}$ and 370kg)

The tonal quality of a music bell is also characterised by its frequency ratios of the partials to the fundamental. The partials of a normal-strike tone include the fundamental frequency of the side strike tone (as the first partial) and other higher partials of the bell. The partials of a side-strike note have the same frequencies of the higher partials of the normal-strike tone (with different peak amplitudes). For latter, the fundamental frequency of the normal-strike note becomes its sub-partial.

The measured fundamental and partial frequencies of the 64 Marquis Zeng Yi's musical bells are presented in Figure 7 in frequency ratios with respect to the corresponding fundamentals of their normal-strike tones. In summary, the frequency ratios of the normal-strike tone are

$$f_{N1,m} : f_{N2,m} : f_{N3,m} = 1 : (2^{1/12})^3 : (2^{1/12})^{12+6}, \quad (1)$$

where $m = 1, 2, ...64$ are the bell orders with $m = 1$ referring to the bell at the left-top corner of Figure 1 and I1.1 of Figure 2. The frequency ratios of the side-strike tone are

$$f_{S0,m} : f_{S1,m} : f_{S2,m} = (2^{1/12})^{-3} : 1 : (2^{1/12})^{12+3}, \quad (2)$$

where $f_{Sn,m} = f_{Nn+1,m}$ for $n = 0, 1, 2$. The ratios for $n > 2$ are not conclusive.

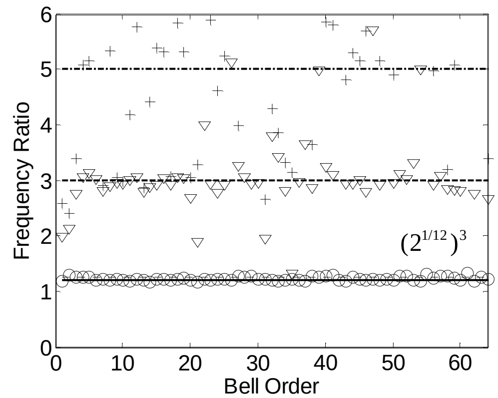


Figure 7. Frequency ratios of 64 Marquis Zeng Yi's musical bells with respect to $f_{N1,m}$ (O: $f_{S1,m} / f_{N1,m}$, $f_{N2,m} / f_{N1,m}$; ∇: $f_{N3,m} / f_{N1,m}$, $f_{S2,m} / f_{N1,m}$; +: $f_{N4,m} / f_{N1,m}$, $f_{S3,m} / f_{N1,m}$)

MUSIC SCALES OF THE BELLS

The measured fundamental frequencies of the normal and side-strike notes also allowed an investigation of the intended design and tuning of music scales of the music bells.

From Figure 7, we can conclude that the frequency ratio between the fundamentals of the side and normal strike notes of each bell is approximately a minor third and noted as

$$\frac{f_{S1,m}}{f_{N1,m}} = (2^{1/12})^3 . \quad (3)$$

Also based on the frequencies of the 64 bells, the ratios of the fundamentals of the adjacent bells (in 9 groups corresponding to their positions starting from the left-top corner of Figure 1) are shown in Figures 8 and 9.

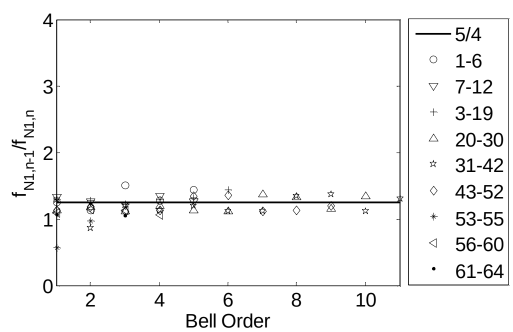


Figure 8. Ratio of the fundamental frequencies of normal-strike tones of adjacent bells

In summary, the frequency ratio of the adjacent bells (in 9 groups) is approximately a major third and noted as

$$\frac{f_{N1,m}}{f_{N1,m+1}} = \frac{f_{S1,m}}{f_{S1,m+1}} = (2^{1/12})^4 . \quad (4)$$

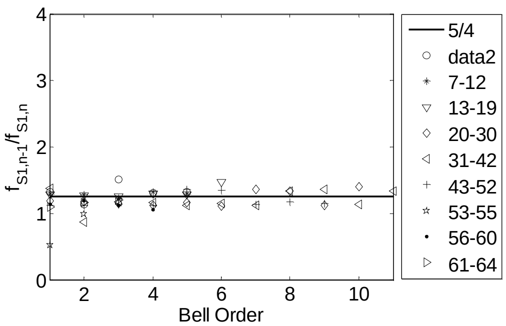


Figure 9. Ratio of the fundamental frequencies of side-strike tones of adjacent bells

The design of the minor third interval between the side and normal strike tones in each bell and of the major third between the adjacent bells allowed a semi tone between the normal strike tone of the adjacent higher (in frequency) bell (m) and the side strike-tone of the $m+1$ bell:

$$\frac{f_{N1,m}}{f_{S1,m+1}} = (\frac{f_{N1,m}}{f_{S1,m}})(\frac{f_{S1,m}}{f_{S1,m+1}}) = 2^{1/12} . \quad (5)$$

This result indicates that the ancient Chinese music bells are capable of producing 12 semi-tones in an octave. Figure 10 is an illustration of the logical combination of the Marquis Zeng Yi's musical bells from the two top rows of group J and K of Figure 2. In the vertical direction of the "bell matrix" in Figure 10, the frequency interval is a minor third, which is determined by the frequency ratio of the normal and side-strike tones of each bell. The diagonal direction of the matrix has a major third interval due to the tuning between adjacent bells. As a result, a clear semi-tone increment of bell notes is achievable in the horizontal direction of the matrix.

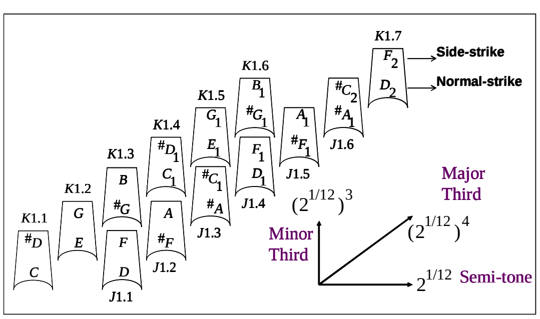


Figure 10. Tonal intervals of two top rows in Marquis Zeng Yi's musical bells (see Figure 2). For convenience of the notation the lowest tone at 365.1 Hz is defined as C

BALANCE OF BELL SOUND

A glance of the bell sizes in Figure 1 indicates that the ancient bell makers might have used scaling rules to achieve the required frequency intervals between the adjacent bells. It is also important to minimise the effort in the bell tuning by designing the correct bell dimensions in the first place. The relative dimensions of Chinese music bells are illustrated in Figure 11 based on the description given in [4]. It appears that all the bell dimensions are scaled with respect to the bell height H .

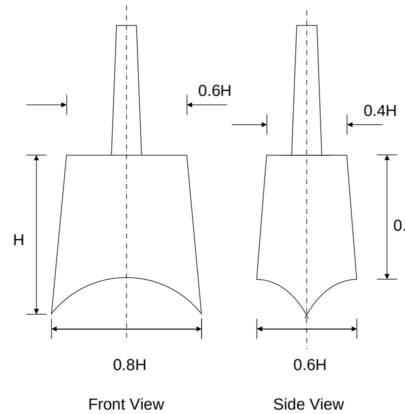


Figure 11. Relative dimensions of Chinese music bells in terms of the bell height H

To rediscover the scaling rules that ancient bell makers employed for frequency tuning, the frequencies of the normal-strike notes of the 45 Marquis Zeng Yi's musical bells (second and bottom rows of Figure 1) are plotted in Figure 12 as a function of bell height.

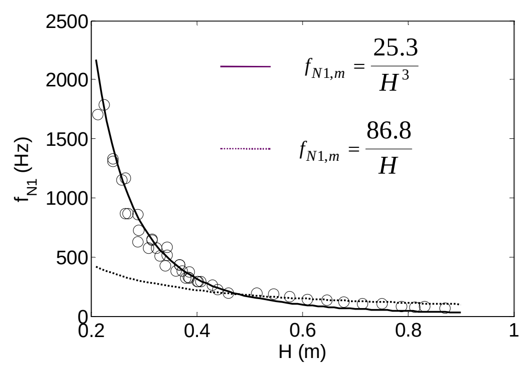


Figure 12. Fundamental frequencies of the normal-strike notes of 45 Marquis Zeng Yi's musical bells as a function of bell height

In Figure 12, the experimental data are fitted by two curves, which suggest that the fundamental frequencies of the music bells were actually scaled with H by two rules. For larger bells with $H \geq 0.42m$, the scaling rule of $f_{N1,m}$ is

$$f_{N1,m} = \frac{86.8}{H} . \quad (6)$$

While for smaller bells with $H \leq 0.42m$, the scaling rule is

$$f_{N1,m} = \frac{25.3}{H^3}\,. \qquad (7)$$

To explain how the bell makers implemented the two scaling rules, it is worth to look at the bell's cross section on the bell's horizontal plane. Among all the bell dimensions, the distribution of the wall thickness of the bell as a function of the angular position appeared to be an important parameter for the fine tuning. The thickness profile was already determined at the design and casting stages. As illustrated in Figure 13, in the circumferential direction of the bell lip, the normal thickness regions are found at angles close to $\theta = [0, \pi/2, \pi, 3\pi/2]$, where for Marquis Zeng Yi's musical bells the evidence of shaping and smoothing was found. These areas are called sound ditches. The bell wall increases up to 1.5 times of the normal thickness in areas close to $\theta = [\pi/6, 5\pi/6, 7\pi/6, 11\pi/6]$, where some evidence of shaping and smoothing was also found. These areas are called sound spines. Along the vertical direction of the bell (see Figure 11), the wall thickness and angular width of the sound spines gradually decrease to the normal thickness and zero respectively when the position changes from the bell lip to half of the bell height.

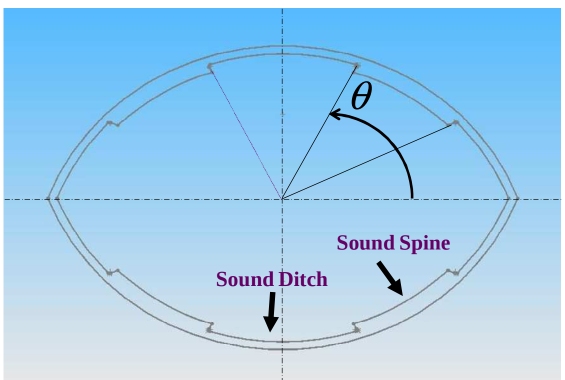


Figure 13. Illustration of bell wall thickness distribution

Figure 14 is the averaged thickness-to-height ratio of the 45 bells measured at the sound spines of the bells and plotted against the bell height. This result suggests that the two scaling rules of the bell's fundamental frequencies in Equations (6) and (7) are achieved by using two different scaling relations between the thickness and height of the bells. As illustrated by the fitted curves in Figure 14, the thickness-to-height ratio for the larger bells ($H \geq 0.42m$) fluctuates around a constant:

$$\frac{t}{H} = 0.014 \qquad (8)$$

while for smaller bells the ratio becomes

$$\frac{t}{H} = \frac{0.0026}{H^2}\,. \qquad (9)$$

In summary the different scaling rules of the bells were practically implemented using the two different scaling relations between t and H.

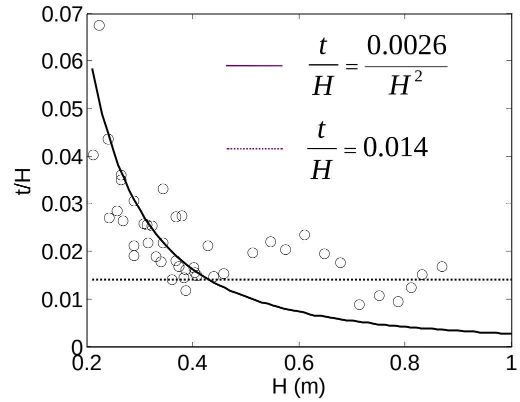


Figure 14. Bell thickness-to-height ratio as a function of bell height

Why did the ancient bell makers use the two scaling rules? The following analysis explains that they did so in order to keep the tonal balance of the music bells. The tonal balance of music bells refers to similar loudness of the bell tones regardless of their location in the frequency scale. In this analysis, the loudness of a bell sound is described by the sound power radiated and the sensitivity of human ear to the associated sound intensity at the fundamental frequency of the bell's tone. Without losing generality, the sound power of the normal-striking note is discussed here. The radiated sound power of the m^{th} bell at $f_{N1,m}$ can be expressed as [5]

$$\Pi_{N1,m} = \rho_o \sigma_{N1,m} < \bar{V}^2_{N1,m} > S_{N1,m} \qquad (10)$$

where $\sigma_{N1,m}$ and $< \bar{V}_{N1,m} >$ are respectively the sound radiation ratio, and time-space averaged modal velocity. $S_{N1,m}$ is the surface area of bell sound radiation and ρ_o is the density of air. Using the finite element and boundary element analysis [6], the sound radiation ratio of the symmetrical mode $(2,0)_S$ is calculated and presented in Figure 15 as a function of non-dimensional wavenumber kH. At the fundamental frequency $f_{N1,m}$ of the m^{th} bell, the sound radiation ratio can be obtained from Figure 15 at $kH = \frac{2\pi f_{N1,m}}{c_o} H_m$. For the 64 music bells analysed here, the value of kH ranges from 1 to 6, where the numerically calculated $\sigma_{N1,m}$ can be fitted by the following analytical curve:

$$\sigma_{N1,m} = 2.3\times10^{-3}\left(\frac{2\pi f_{N1,m} H_m}{c_o}\right)^3. \qquad (11)$$

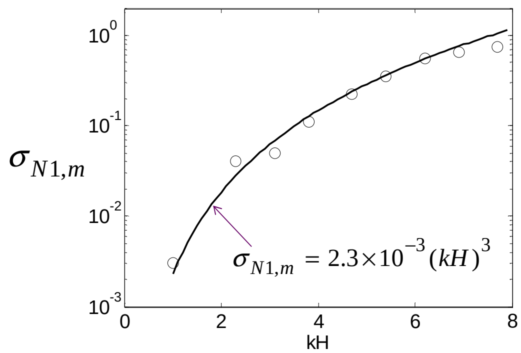


Figure 15. Sound radiation ratio of the $(2,0)_S$ bell mode

Dimensional analysis indicates that the surface area of bell sound radiation is proportion to H_m^2. Using Equations (10), (11) and (6), the sound power ratio between adjacent large bells (increasing of the bell order m corresponds to decrease of the fundamental frequency) is derived as

$$\Pi_{m+1} / \Pi_m = 2^{2/3} \frac{<\bar{V}^2_{N1,m+1}>}{<\bar{V}^2_{N1,m}>}. \qquad (12)$$

For small bells, this ratio becomes

$$\Pi_{m+1} / \Pi_m = 2^{-4/9} \frac{<\bar{V}^2_{N1,m+1}>}{<\bar{V}^2_{N1,m}>}. \qquad (13)$$

Assuming an equal time-space averaged modal velocity of all the bells, Equation (12) demonstrates that the sound power level of the larger bells increases by 2dB for each increment of the bell order m, which corresponds to a decrease in frequency by a major third. On the other hand, Equation (13) suggests that the sound power of smaller bells increases by 1.3dB for each decrease of m corresponding to an increase of a major third.

In Figure 16, the relative sound power of the normal-strike tone of the music bells and the A-weighting correction curve are presented. The purpose of the scaling rule for the largest bells becomes apparent when the increase in sound power and the decrease in the sensitivity of human ear with decreasing of frequency of the bells are viewed together. Because of this rule, the increase in low frequency sound power compensates the reduced sensitivity. As a result, the loudness of low frequency bell sound becomes more balanced. Should this scale rule be also applied to smaller bells, the sound power from higher frequency bells would have had diminishing values with the increasing of the bell tones.

With the second scaling rule for the small bells, the radiated sound power of the bells increases slowly with the increase of bell tones, which effectively avoids the reduction of bell's sound power in the higher frequencies.

Other bell parameter such as bell's damping and impact force during striking may affect the ratio of radiated sound power between adjacent bells. Consequently, the assumption of the equal time-space averaged modal velocity may not hold. Future research on the ratio of modal velocities of adjacent bells may provide a better description of the power ratios of the small bells. The fact that structural damping increases with frequency may set up a limit for the sound power increment of small bells (see Equation (13)).

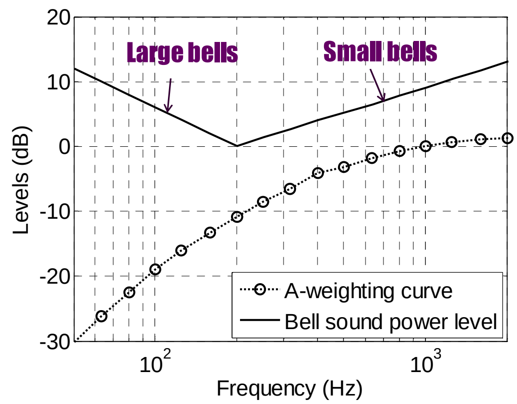


Figure 16. A-weighting curve and relative bell sound power level

CONCLUSIONS

This study on the acoustical properties of ancient Chinese music bells provides the following understandings towards the general features of the entire set of bells as a musical instrument:

(1) The minor third frequency interval between side and normal-strike tones of individual bells and the major third interval between adjacent bells provide 12 semi-tones per octave for music performance of the bells.

(2) Both normal and side strike notes of the bells have stable tonal quality as the fundamental components dominate the entire duration of bell sound radiation. The first two partials of normal-strike note are respectively a minor third $(2^{1/12})^3$, and an octave and a diminished fifth $(2^{1/12})^{12+6}$ above the fundamental. The side-strike note has a sub-partial of a minor third lower $(2^{1/12})^{-3}$ and a first partial of one octave and a minor third higher.

(3) The major third frequency interval between adjacent bells is realized by two scaling rules. The scaling rules were implemented by using two different thickness-to-height ratios for different bell heights.

(4) The reason for using the two scaling rules is most likely for the purpose of achieving more balanced tones among bells of different sizes.

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