DETERMINING THE STATIC STIFFNESS OF STRUCTURES: A COMPARISON OF METHODS INVOLVING MEASUREMENT OF DISPLACEMENT AND MEASUREMENT OF ACCELERATION RESPONSE

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Abstract

Two different methods of estimating the static stiffness of structures are compared. In the first approach, the structure is loaded with a known quasi-static force and its displacement is measured. The second approach is based on impact tests with a rubber tipped hammer where the dynamic response of the structure is determined and the static stiffness is obtained by extrapolating the receptance spectrum towards zero Hertz. This paper compares the results of both methods from measurements taken on an existing bridge, and discusses the relationship between the measured static stiffness using the force-displacement method and the dynamic approach based on impact tests.

1. Introduction

The static stiffness of a structure is an important design parameter as knowledge of the in-situ static stiffness can be used to calibrate models to increase their accuracy. However, despite the importance of this parameter, practical in-situ methods of determining the static stiffness of existing structures are limited; particularly because large, engineered structures require comparatively large loads to achieve measurable deflections. Often, the application of appropriate loads is difficult and requires considerable planning and effort.

The impact tests (also known as mobility tests, dong tests or modal tests) require comparatively little effort and the necessary instrumentation effectively boils down to an instrumented hammer, accelerometer and data recorder (ie instrumentation often owned in-house by specialist firms). The two methodologies followed to estimate the static point load stiffness of a structure are:

- Applying a static load with a known magnitude, and measuring the corresponding displacement of the structure. The load applied can then be divided by the measured displacement to yield the static stiffness. In the presented case, the load did not resemble a point load and an intermediate step utilising a calibrated FE model had to be employed.

- Performing impact tests to determine the dynamic response of the structure. In this paper the acceleration response relative to the input force was measured and then analysed in the frequency domain to determine the receptance spectrum (the receptance relates displacement to force) and its inverse, the dynamic stiffness.

1.1 Test structure

The tested structure is a bridge deck, as shown in Figure 1. The test span is 23 m long and consists of six precast prestressed concrete I-girders with an approximate height of 1.1 m. The girders are acting compositely with the in-situ cast, reinforced concrete deck slab which is 0.15 m thick.

The test span is an end span. As such the I-girders on one end are resting on bearings supported on the abutment. At the other end, the superstructure is resting on a headstock on one central pier. The central section of the pier is a circular column (nominal diameter 1.7 m), supporting a tapered headstock. Bearings are in between the girders and the abutment and headstock. The load application points on the deck (tyre footprints and point load application points) are shown in grey on the right hand view, Figure 1.

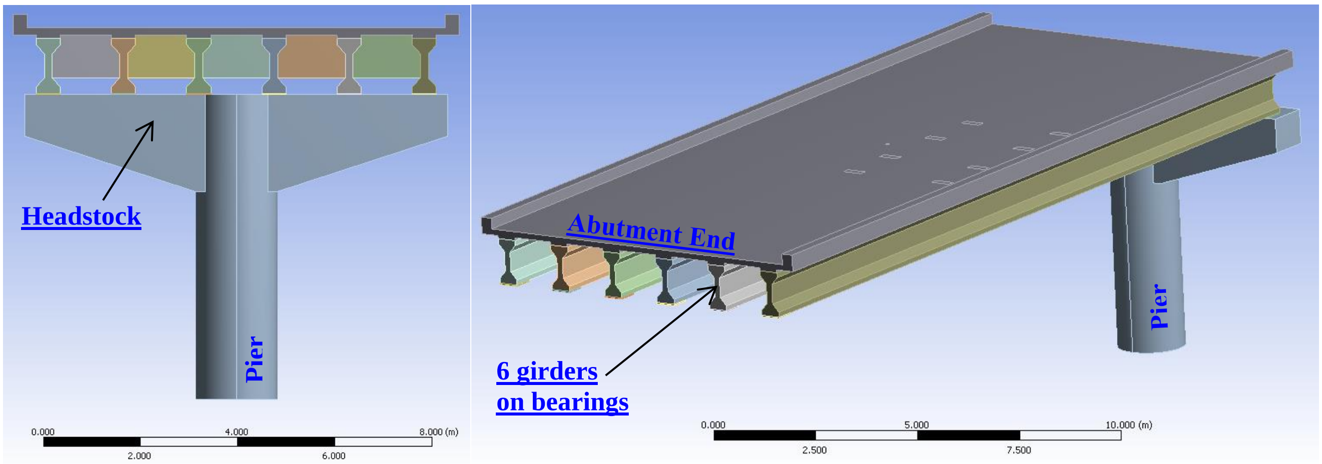

Figure 1. Tested bridge deck

2. Quasi-Static Tests

2.1 Measurements

For the quasi-static tests the structure was loaded with a test vehicle with known axle loads. The total vehicle load was 47 t and the test vehicle's maximum axle spacing was 5.6 m, i.e. shorter than the test span. The test vehicle lane-crawled over the bridge (two lane bridge) and strain and deflections were recorded continuously. The measured variables when the test vehicle was located at midspan were extracted and used for calibrating a Finite Element model of the test span. Table 1 provides a summary of the measured variables.

Table 1. Measurements

Measurement Variable	Location	Measured
Vertical deflection$^{(1)}$	Outside girder, vehicle lane, midspan	-4.66 mm
	Outside girder, non-vehicle lane, midspan	-0.85 mm
Bearing compression$^{(2)}$	Bearings on the headstock, vehicle lane	140 to 170 μm
	Bearings on the headstock, non-vehicle lane	10 to 80 μm
Tensile strain$^{(3)}$	Midspan girders, vehicle lane	55 to 75 με
	Midspan girders, non-vehicle lane	10 to 40 με
Fundamental Bending Frequency$^{(4)}$	Midspan (both centreline and kerb)	5.75 Hz

(1) Deflection relative to ground measured with a Linear Variable Differential Transformer (LVDT).

(2) Relative movement of girder underside to headstock next to bearing measured with an inductive proximity probe.

(3) Strain measured on the underside of the girders.

(4) From impact tests.

2.2 Calibrating FE model

A Finite Element model of the bridge was built in the 3D CAD package Creo and imported into ANSYS. The model domain is shown in Figure 1 and comprised the pier, bearings on the pier and the abutment and the superstructure. The model was constrained such that the bearing pads on the abutment and the bottom of the pier were unable to translate in any direction.

The model was meshed with second order solids. The rather coarse mesh was chosen to enable speedy analysis keeping in mind that deflections rather than stresses were used primarily for calibrating the FE model with results. The element count for the model was 7,729.

The FE model was loaded with the nominal vehicle wheel loads at locations consistent with the vehicle's centre at midspan (refer to 6 rectangular wheel footprints, Figure 1 right hand view). Linear analyses procedures were used and the deflections at the nodes coinciding with the deflection measurements were extracted and compared against the measured values in Table 2. The main tuning parameter was the effective Young's modulus of the reinforced girders.

Figure 2 shows the predicted deflections (black on cyan) for the tuned model. The predicted deflections and the measured deflections are also presented side-by-side in Table 2 and good agreement is observed. The calculated strains and bearing deflections also closely matched the measured values.

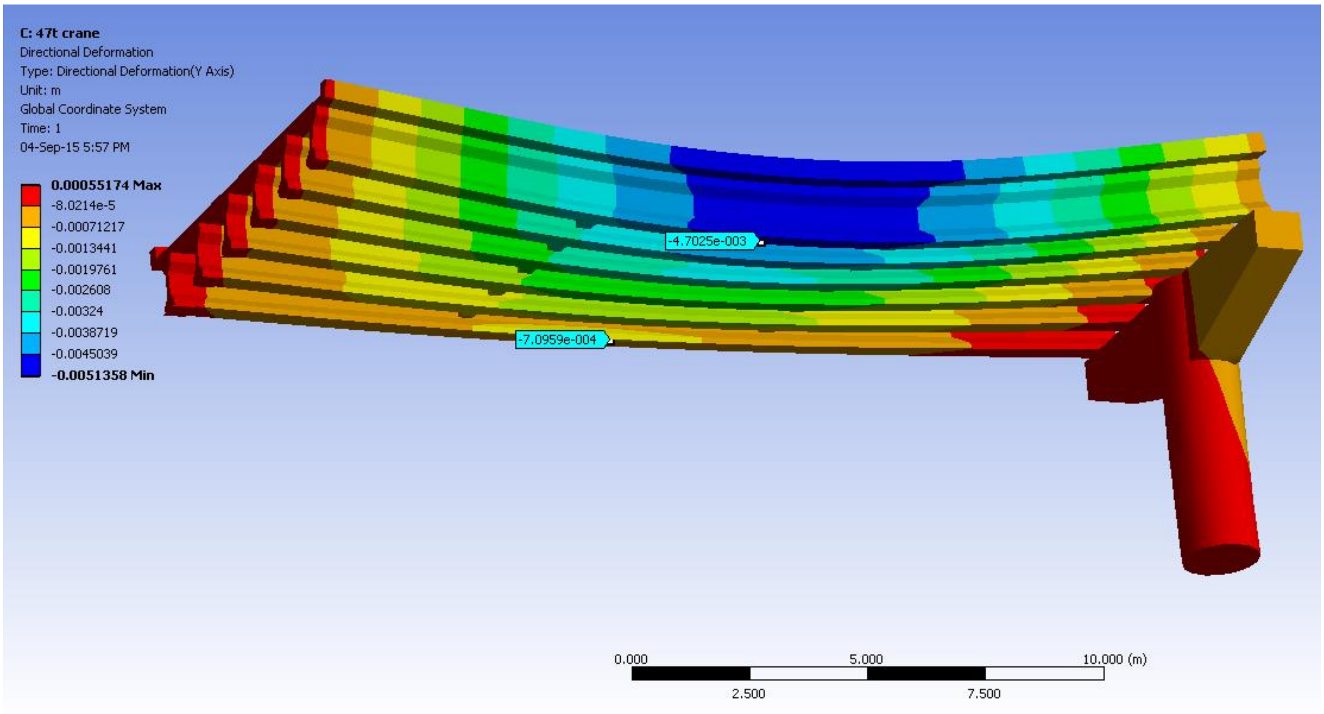

Figure 2. Vertical deflections for test vehicle load as predicted in Ansys (black on cyan background)

Table 2. Side-by-side comparison of the measured vertical deflections compared to the predicted vertical deflections

Measurement Variable	Location	Measured	Modelled
Vertical deflection	Outside girder, vehicle lane, midspan	-4.66 mm	-4.70 mm
	Outside girder, non-vehicle lane, midspan	-0.85 mm	-0.71 mm
Bearing compression	Bearings on the headstock, vehicle lane	140 to 170 μm	100 to 130 μm
	Bearings on the headstock, non-vehicle lane	10 to 80 μm	40 to 70 μm
Tensile strain	Midspan girders, vehicle lane	55 to 75 με	70 to 90 με
	Midspan girders, non-vehicle lane	10 to 40 με	15 to 45 με

2.3 Static Stiffness

The calibrated model was subsequently loaded with a vertical unit point load at midspan in the centre and at midspan at the edge. The chosen positions correspond with the impact test locations.

The static point stiffness was obtained by dividing the applied force by the predicted deflection at the load application point. The static stiffnesses were calculated to be:

- Midspan, centreline: 124 MN/m
- Midspan, kerb: 58 MN/m

In the centre the stiffness is more than twice the stiffness on the edge. This is primarily due to rotation of the headstock. This difference between centreline and kerb stiffness in vertical direction would be more pronounced on the spans away from abutments.

For comparison, dividing the test vehicle's equivalent force (47 t) by the average midspan deflection of 2.8 mm, gives an 'average' or 'smeared out' stiffness of 168 MN/m. This smeared out stiffness is greater than the midspan centreline and kerb values of 124 MN/m and 58 MN/m, respectively, because the contributions due to pier rotation on either side of the bridge cancel (vehicle lane going down, non-vehcile lane going up), yielding a stiffness based on deck bending and bearing deflection, only. Using the minimum and maximum midspan deflections gives a stiffness range of 545 MN/m to 99 MN/m, respectively.

3. Dynamic Tests

The possible scope of dynamic, in-situ tests of structures is vast; ranging from single channel measurements for estimating vibration amplitudes and dominant frequencies to multi-channel measurements for determining mode shapes and modal parameters.

The dynamic properties of two points on the bridge deck were determined with two channel force-response measurements. A vertical force was exerted with an instrumented sledge hammer and the vertical acceleration response next to the impact location was measured with an accelerometer. Force and acceleration were measured simultaneously. This type of test is often referred to as impact test, mobility test or dong test and discussions of this method can be found in the provided references [1], [2] and [3] as well as in most textbooks on structural dynamics.

The objective of the impact tests was to determine the modeshapes of the bridge, and not to determine the static stiffness of the deck. As such, the selected instrumentation was not optimised to measure the vibration response of the structure at low frequencies. If the objective had been to determine the static stiffness of the deck, accelerometers with better low-frequency performance characteristics would have been selected. Additionally, a softer tip for the impact hammer would have been used, which would have resulted in the hammer blows providing more energy at low frequencies.

3.1 Signal processing

In the dynamic tests the bridge deck was impacted at midspan on the centreline and close to the kerb. The force input and the acceleration response immediately adjacent to the impact location were recorded simultaneously. The accelerance spectra were calculated using six second windows and averaged over four impacts, without utilising a windowing function. Subsequently, the accelerance spectra were converted to mobility and receptance spectra through division by the circular frequency. As the inverse of the receptance spectrum is the dynamic stiffness spectrum, the dynamic stiffness spectra were easily calculated, and are presented in Figure 3 in units of MN/m.

3.2 Results

The dynamic stiffness spectra for the centreline location and the kerb location are presented in Figure 3 by thick blue and red lines, respectively. Results based on individual impacts contributing to the average spectrum are shown as thin lines. The magnitude is shown in the top graph and the coherence in the bottom graph. The following observations are made:

- The reduction in dynamic stiffness at approximately 5.8 Hz is due to the underlying bending mode. The dynamic stiffness is similar at both impact locations which is consistent with the bending mode shape.
- Excellent coherence is observed at frequencies greater than 4 Hz. At lower frequencies, the coherence drops off. This is also evidenced by greater fluctuations between individual impacts.
- The dynamic stiffness reduces with reducing frequency below approx. 3Hz. This is due to 1/f instrumentation noise and therefore there is no asymptotic tendency of the dynamic stiffness.
- For both the kerb and centreline location there are distinctive coherence drops at approximately 3 Hertz. Using these drop offs (indicated by the blue and red arrows in Figure 3) to extrapolate towards zero Hertz results in static stiffness values of approximately:
    - 90 MN/m at the midspan, centreline
    - 50 MN/m at the midspan, kerb

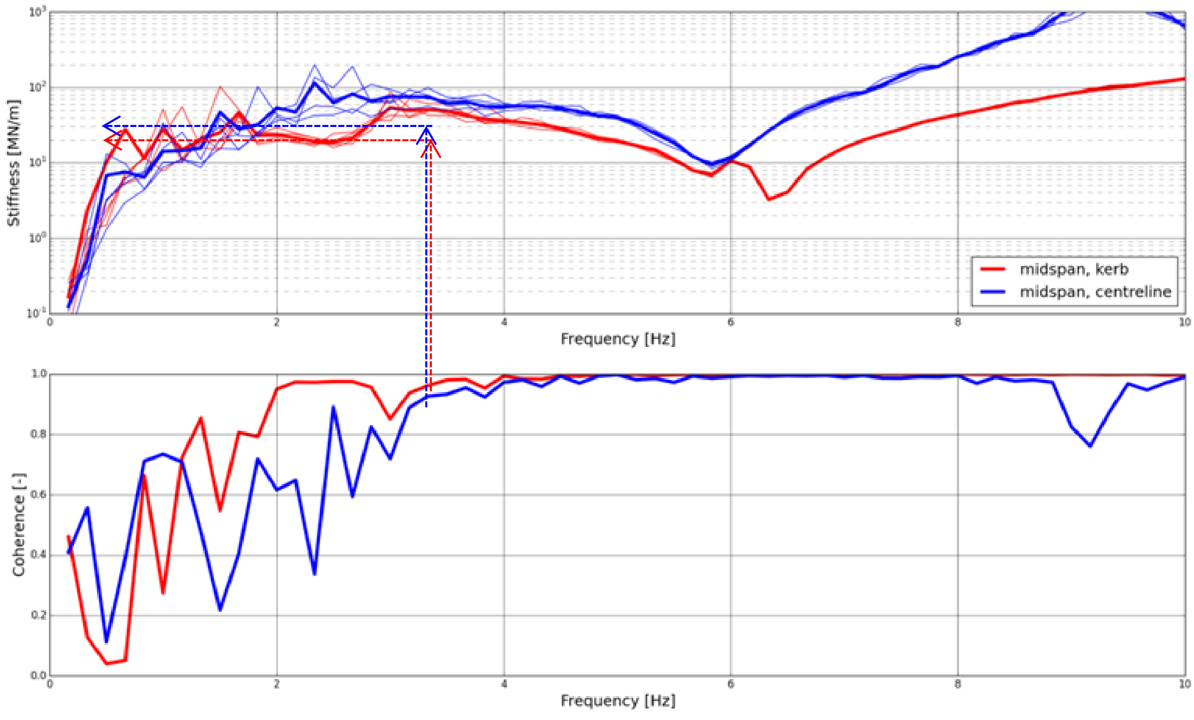

Figure 3. Dynamic stiffness magnitudes (top) and coherence (bottom)

4. Discussion and Conclusions

Two completely different approaches were used to estimate the static stiffness of a bridge deck. Based on the impact test method, the static stiffness:

- Estimated at the centre-line is 1.4 times lower than based on the quasi-static vehicle run.
- Estimated at the kerb is 1.2 times lower than based on the quasi-static vehicle run.

The agreement between the methods is reasonable particularly when considering that the impact tests were not optimised for estimating the static stiffness. Another potential reason contributing to the discrepancy could be nonlinearities. For the quasi-static tests the maximum deflections were 4.7 mm, while the deflections from the force input were typically 0.03 mm in the impact tests.

References

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