Sound absorption of a soft medium embedded with hard spheres

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ABSTRACT

Sound absorption by an acoustic coating comprising a hexagonal lattice of hard spherical inclusions embedded in a soft elastic medium is analytically and numerically investigated. The analytical approach approximates each layer of inclusions in the direction of sound propagation as a homogenised layer incorporating local resonance of the inclusions and multiple scattering of waves between the inclusions in the lattice. The analytical results for sound absorption are in excellent agreement with finite element simulations that exactly model the geometric and material properties of the coating.

1 INTRODUCTION

Acoustic coatings for maritime applications comprise a soft elastic medium embedded with a lattice of voided and/or hard inclusions (Meresse et al., 2015; Sharma et al., 2019a). Voided and hard inclusions in a soft medium respectively exhibit monopole resonance and dipole resonance (Sharma et al., 2019b, 2020). Strong attenuation of sound occurs at these subwavelength resonances due to the conversion of acoustic waves into shear waves which are dissipated in the soft host medium with high shear damping. An acoustic coating comprising hard inclusions has specific practical importance due to consistent acoustic performance under hydrostatic pressure (Meresse et al., 2015). In this work, sound absorption of a plane acoustic wave at normal incidence by a hexagonal lattice of hard spheres in a soft elastic medium, as shown in Fig. 1(a), is presented.

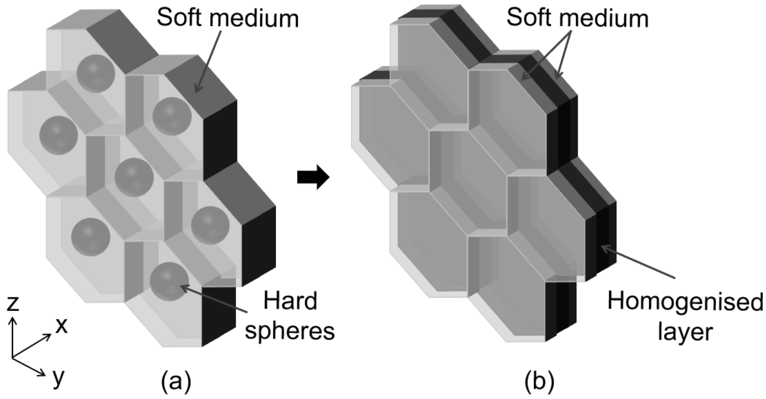
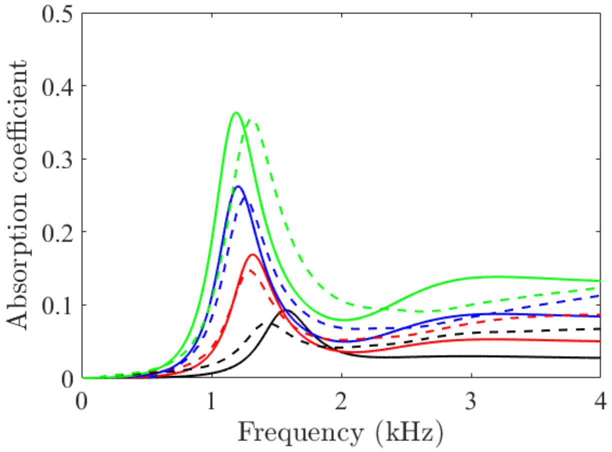


Figure 1: Schematic diagram showing (a) a hexagonal lattice of hard spheres embedded in a soft medium, (b) the layer of inclusions approximated as a homogenised layer with effective properties. The coating is of infinite extent in the yz plane and is subject to acoustic plane wave excitation in the x direction.

2 ANALYTICAL FRAMEWORK

For the analytical model, each layer of spheres in the direction of sound propagation is approximated as a homogenised layer with effective longitudinal modulus, effective density and effective thickness, as shown in Fig. 1(b). The layer approximation reduces the three dimensional scattering problem to one dimension. The effective thickness for a layer of spheres arranged in a hexagonal lattice is approximated as the blockage length due to potential flow around a row of hard spheres in a circular tube with the same cross-sectional area as the hexagonal unit cell (Cai and Wallis, 1992). The effective density and longitudinal modulus are derived in terms of the material properties of the host medium and inclusions, radius of the inclusions, lattice spacing and the monopole and dipole scattering amplitudes of a hard sphere in a soft medium (Waterman and Truell, 1961). At high filling fraction of scatterers, multiple scattering of waves between inclusions becomes important. Multiple scattering effects are accounted for using expressions for the added mass and drag coefficients of a sphere at the centre of a tube. The reflected and transmitted pressures are computed using the standard formulae for a layered medium (Brekhovskikh, 1960). The absorption coefficient is then calculated using the reflected and transmitted pressures.



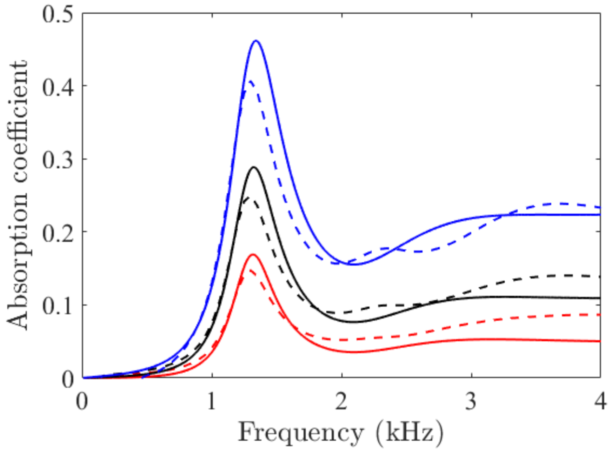


Figure 2: Absorption coefficient of one layer of hard spheres in a soft medium for inclusion radius of 0.75 cm (black lines), 1 cm (red lines), 1.25 cm (blue lines) and 1.5 cm (green lines), obtained analytically (solid lines) and numerically (dashed lines).

Figure 3: Absorption coefficient of one layer (black lines), two layers (red lines) and four layers (blue lines) of hard spheres in a soft medium, obtained analytically (solid lines) and numerically (dashed lines).

3 RESULTS AND DISCUSSION

The densities of the elastic medium and spherical inclusions are 1000 kg/m^3 and 7890 kg/m^3, respectively. The longitudinal and shear wave speeds in the elastic medium are 1500(1 + 0.01i) m/s and 78.31 + 11.49i m/s. The distance from the water on both incidence and transmission sides to the centres of the spheres within the layer is 5 cm. The spacing between spheres in the lattice is 5 cm. For validation of the analytical framework, a finite element model using COMSOL Multiphysics (v5.5) was developed. Only a unit cell comprising a single spherical inclusion at the centre of a hexagonal prism made of soft material was numerically simulated. A lattice of inclusions was accounted for by applying periodic boundary conditions on the rectangular faces of the unit cell. Figure 2 presents the absorption coefficient of a soft medium embedded with a single layer of hard spheres of different radius. Multiple scattering of waves by the lattice of spheres enhances the effect of damping around the dipole resonance frequency, at which the incident sound waves are converted into shear waves and subsequently absorbed due to high shear damping. With increasing proximity of spheres associated with increasing radius values, enhanced multiple scattering effects and higher volume fraction of scatterers results in greater sound absorption around dipole resonance. Figure 3 presents the absorption coefficient for one, two and four layers of spheres in the elastic medium. In each layer, spheres of radius 1 cm are arranged in a hexagonal lattice with a lattice spacing of 5 cm. The distance between layers in the direction of sound propagation is also 5 cm. The frequency of dipole resonance of the spheres is not significantly affected by the additional layers of scatterers. However, the sound absorption increases as the number of layers is increased arising from greater generation and dissipation of shear waves. Results obtained using the analytical model are in excellent agreement with numerical simulations.

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