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Wavepacket Coupling in Screeching Twin-Jets

Petrônio A. S. Nogueira, Michael N. Stavropoulos and Daniel M. Edgington-Mitchell

Laboratory for Turbulence Research in Aerospace and Combustion, Monash University, Clayton, Australia

ABSTRACT

An analysis of the symmetry-locking mechanism in screeching twin-jet systems dominated by axisymmetric modes is performed in this work. The different waves supported by the flow in a range of jet conditions are obtained by means of a twin-jet vortex sheet model, which considers the shear layer as an infinitesimal region. Analysis of the bands of existence of upstream waves in the flow suggest that the jet separation greatly affects the ability of the flow to support anti-symmetric screech modes, while symmetric modes remain relatively unaffected by this parameter. Comparison with acoustic data shows that most tones lie in the frequency bands of existence of guided jet modes, supporting the hypothesis that resonance is closed by these waves. The dominant symmetry for each condition is obtained by means of a symmetry-imposed spectral proper orthogonal decomposition of schlieren data, which provides both mode shapes and energies of the most amplified coherent structures in the flow. Overall, it is shown that symmetric modes are more energetic for very low spacings, and no clear dominance is found for large spacings.

1 INTRODUCTION

Screech tones, sharp acoustic peaks generated by imperfectly expanded supersonic jets, were first identified in the seminal work of Powell (Powell, 1953). Since then, several efforts have been done to further understand the mechanism of generation of these tones (Raman, 1999). As highlighted in previous reviews (Edgington-Mitchell, 2019), screech is caused by a feedback loop involving the downstream-travelling Kelvin-Helmholtz mode (Jordan and Colonius, 2013), responsible for the generation of large-scale coherent structures in the flow, and an upstream wave. Recent works (Edgington-Mitchell et al., 2018, Gojon et al., 2018) suggested that this upstream wave is actually a guided jet mode – a wave that is supported by the flow only at specific frequencies. Prediction models using this assumption lead to good agreement with experimental results, supporting this hypothesis (Mancinelli et al., 2019). These previous works also show that the characteristics of both downstream- and upstream-travelling waves can be studied by means of tools based on linear stability analysis, such as the vortex sheet model.

The resonance phenomenon associated with screech can be severely modified in multi-stream jets (Raman et al., 2012, Knast et al., 2018). Now, screech tones will also be a function of spacing, and dominant structures in the jets are also subject to coupling following different symmetries. These dynamics can also generate new tones in the spectrum, and substantially increase screech amplitude. Due to the underlying complexity of this geometry, the analysis of the waves supported by the flows is usually based on single jet results, which provides no clarity on the reasons for coupling. Exception is found in some recent works (Morris, 1990, Du, 1993, Rodriguez et al., 2018, Stavropoulos et al. 2021), where linear stability models were used to analyse the characteristics of the different waves supported by the flow. Still, most of these works were focused on either subsonic or ideally expanded jets; thus, no clear application for screeching jets could be obtained. In (Stavropoulos et al., 2021), the authors analysed the screech phenomenon in twin jets using linear stability and acoustic data, but no confirmation concerning the symmetry selected by the flow for each condition could be obtained from the experimental results. Up to this date, it is not clear which mechanism underpins symmetry-locking in screeching twin-jets.

The present work focuses on the study of coupled round twin-jet systems at low supersonic Mach numbers. By means of a twin-jet vortex sheet (TJVS) model, predictions of which symmetry will be dominant as a function of spacing are performed. Predicted regions of existence of screech tones from this model are compared to acoustic data, which provides the frequencies of the peaks without any indication of the symmetry selected by the system. Symmetry selection is assessed by means of a symmetry-imposed spectral proper orthogonal decomposition (sym-SPOD) of schlieren data, which allows us to extract the most energetic coherent structures around the screech tone. Comparing the energies of modes related to both symmetries, an estimate of the dominance of each symmetry can be obtained, and compared to the TJVS model. Such analysis also allows us to evaluate coupled modes outside the screech frequency. The paper is organised as follows: First, in section 2, the twin-jet vortex sheet model is detailed. Afterwards, both acoustic and schlieren experiments are described in section 3, as well as the process to obtain sym-SPOD modes. In section 4, predictions of symmetry dominance based on the TJVS model are compared with both acoustic and sym-SPOD. The paper is closed in section 5 with a summary of the main results.

2 Twin-jet vortex sheet model

The present formulation is based on the twin-jet vortex sheet model as derived by (Morris, 1990). The model comprises two jets of diameter D, with centres distant by S (normalised by the jets diameter), and whose shear layers are considered to be infinitesimal, as shown in Figure 1(a). Following this assumption and considering normal modes in both time and streamwise direction, the compressible Euler equations can be reduced in both inner (i) and outer (o) regions of the jets to a single equation for the pressure fluctuations p as

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \lambda_{i,o}^2\, p = 0 \tag{1}$$

with

$$\lambda_i^2 = \sqrt{k^2 - \frac{1}{T}\,(\omega - M\,k)^2}, \tag{2}$$

$$\lambda_o^2 = \sqrt{k^2 - \omega^2}. \tag{3}$$

In the equations above, M is the acoustic Mach number, T is the jet to free-stream temperature ratio, k is the streamwise wavenumber, and ω is the frequency. Following Morris (Morris, 1990), both inner and outer solutions may be expressed as an infinite sum of Bessel functions. By imposing matching conditions at the boundary of the jets, one can write the following dispersion relation for a twin-jet system

$$\sum_{m=0}^{\infty} A_m\,[a_{nn}\,\delta_{mn} \pm (-1)^m\,c_{mn}] = 0, \tag{4}$$

with

$$a_{nn} = \frac{1}{\left(1-\frac{kM}{\omega}\right)^2} - \frac{1}{T}\frac{\lambda_o}{\lambda_i}\frac{K_n'\left(\frac{\lambda_o}{2}\right)I_n\left(\frac{\lambda_i}{2}\right)}{I_n'\left(\frac{\lambda_i}{2}\right)K_n\left(\frac{\lambda_o}{2}\right)}, \tag{5}$$

$$c_{mn} = (-1)^n\varepsilon_n\,[K_{m-n}(\lambda_o S) \pm K_{m+n}(\lambda_o S)]\left[\frac{1}{\left(1-\frac{kM}{\omega}\right)^2}\frac{I_n\left(\frac{\lambda_o}{2}\right)}{K_n\left(\frac{\lambda_o}{2}\right)} - \frac{1}{T}\frac{\lambda_o}{\lambda_i}\frac{I_n'\left(\frac{\lambda_o}{2}\right)I_n\left(\frac{\lambda_i}{2}\right)}{I_n'\left(\frac{\lambda_i}{2}\right)K_n\left(\frac{\lambda_o}{2}\right)}\right], \tag{6}$$

where δ_{mn} is the Kronecker delta, ε_n is equal to 0.5 if $n = 0$, and $\varepsilon_n = 1$ otherwise, I_n and K_n are the modified Bessel functions of the first and second kinds, and the prime ($'$) denotes their derivatives. The equations above must be solved to allow for inhomogeneous solutions ($A_m \neq 0$) for the system denoted in equation (4); thus, the determinant of the matrix defined by the coefficients inside the brackets in equation (4) must be zero, which leads to the solution of the dispersion relation for a given pair (ω, k). One should note that four solutions are allowed by this dispersion relation, each one following a symmetry of the system (see (Morris, 1990, Rodriguez et al., 2018)), selected by the $\pm$ signs in the equations. Here, the solutions are sought numerically using Matlab.

The TJVS model is used to evaluate the characteristics of the different waves supported by the flow. In particular, we are interested in the bands of existence of upstream-travelling guided jet modes, which were shown to be responsible for closing the screech resonance mechanism in the single jet case (Edgington-Mitchell et al., 2018); the difference in the bands of existence of these waves for different configurations may shed light on the particular coupling configuration followed by the jets. Since the focus in the present work is in the low-Mach number region, the flow is expected to be dominated by axisymmetric modes (azimuthal wavenumber $m = 0$) in each jet; thus, the analysis will be focused on modes related to $m = 0$. These modes can couple in a symmetric (even) or anti-symmetric (odd) configuration (thus, only two of the four solutions exist for this case), and the characteristics of the relevant waves following these two symmetries may be one of the key ingredients for symmetry locking.

3 EXPERIMENTAL METHODOLOGY

Both the schlieren and acoustic data were acquired for two axisymmetric converging nozzle (diameter $D = 10mm$) at the Laboratory for Turbulence Research in Aerospace and Combustion, in the supersonic jet facility. The present dataset was explored in a previous work (Knast et al., 2018), and further details about the facility and the nozzles can be found in the cited reference. Acoustic measurements were performed using a G.R.A.S. Type 46BE 1/4 in. pre-amplified microphone with a frequency range of $4\ \mathrm{Hz}-100\ \mathrm{kHz}$. The microphone was placed at a distance of $R = 100D$ from the centre of the first nozzle, at $\theta_1 = 180^{\circ}$ (see Figure 1(a)). The complete data set comprises 5×10^5 samples, recorded at $200\mathrm{kHz}$. For the Welch method, 4096 points were used in the Fourier transform, with 75% overlap, leading to 485 blocks. It is important to highlight that this facility is not anechoic. Still, it can be expected that the peak screech frequency will be well captured in the present case. All results are presented as a function of Strouhal number $St = fD/U_j$, where U_j is the ideally expanded jet velocity.

The present schlieren configuration captures the path-integrated streamwise density gradient ($\partial\rho/\partial x$), which will allow us to identify coherent structures in the flow. The time-resolved luminous intensity fields were obtained using a Shimadzu HPV-1 camera with a resolution of 312×260 pixels. The camera captures 102 images at an acquisition speed of $250\ \mathrm{kHz}$. For each configuration, the experiment was repeated five times, leading to a total of 510 images for each Mach number and interjet distance. A sample image for nozzle pressure ratio $NPR = 2.2$, $S/D = 2$ is shown in Figure 1(b). The images were imported to Matlab, allowing for a definition of a numerical value q for the luminous intensity in each picture, which was decomposed into mean $\bar{q}$ and fluctuation fields (q'). The latter was then decomposed into even (e, symmetric) and odd (o, anti-symmetric) fields in each block of measurement as

$$q'_e(x,y,t) = \frac{q'(x,y,t)+q'(x,-y,t)}{2}, \tag{7}$$

$$q'_o(x,y,t) = \frac{q'(x,y,t)-q'(x,-y,t)}{2}, \tag{8}$$

where (x, y) are the streamwise and cross-flow coordinates normalised by the diameter, following Figure 1(a). Afterwards, a spectral proper orthogonal decomposition (SPOD) was performed in the symmetry-decomposed data, following (Towne et al., 2018). Due to the limited amount of images in each block of the Welch method (102), the discretisation in frequency here is poor, and certainly insufficient to characterise the spectrum perfectly. However, as the focus is to analyse the dominance of each coupling symmetry, a comparison between the energy of dominant even and odd modes can still lead to physical conclusions. In the present case, the number of blocks is restricted to 5 (which is the number of realisations), which leads to only 5 modes in the SPOD. By decreasing the number of images to $n_{fft} = 64$ in each block, and including a 75% overlap, 15 modes are obtained, but this leads to a loss in the already poor frequency resolution ($\Delta f = 2.45\mathrm{kHz}$). A comparison between the shapes/energies of the structures of the leading modes for both cases ($n_{fft} = 102$ and 64) did not lead to significant changes; thus, we decided to keep $n_{fft} = 102$. For the present case, the data was acquired for $NPR = 2$, 2.1, and 2.2, for $S/D = 2$, 3, and 6.

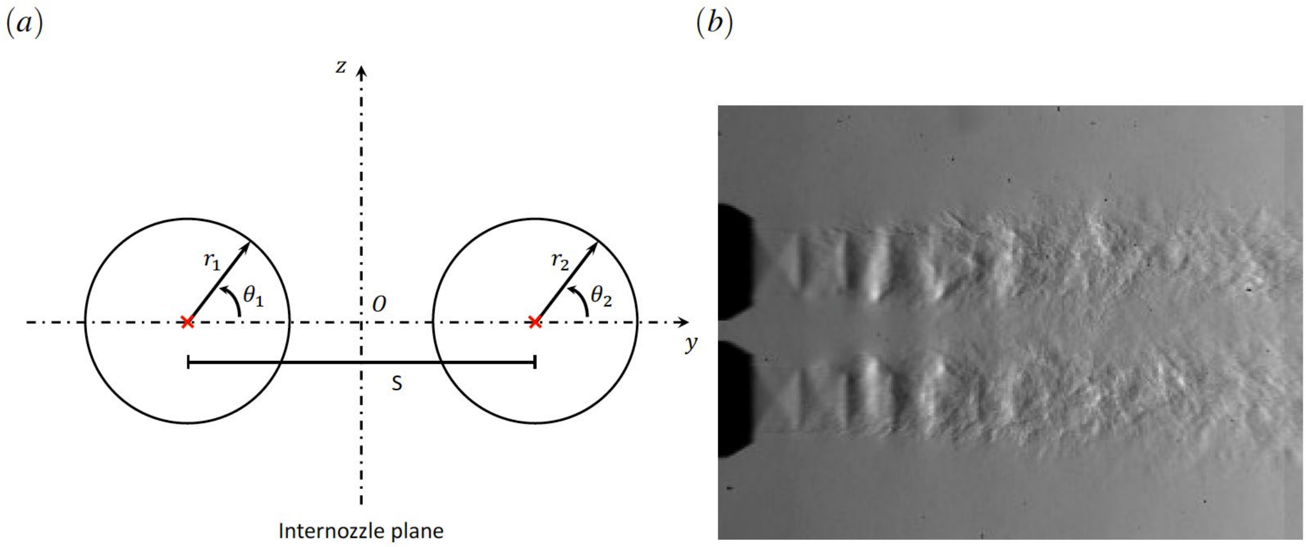

Figure 1: Sketch of the present configuration: (a) shows the coordinates system used in the present analysis and (b) shows a sample schlieren image for $NPR = 2.2$ and $S/D = 2$.

4 RESULTS

4.1 Coupling predictions from VS model

Recent works have shown that the upstream-travelling guided jet mode is responsible for closing the screech resonance loop in underexpanded jets (Edgington-Mitchell et al., 2018, 2021).The existence of the same mode in twin-jets was shown by Du (Du, 1993); a recent study (Nogueira and Edgington-Mitchell, 2021) suggests that this wave is also a key element in the resonance of twin-jet systems at high NPR. In Figure 2, the acoustic spectra are plotted as a function of NPR for different values of spacing. The plots also show the bands of existence of symmetric and anti-symmetric guided jet modes for each case. The first clear feature from Figure 2 is that all axisymmetric (A1/A2) tones from the spectrum lie in the region of existence of symmetric guided modes. It is also clear that, for low spacing, the bands of existence of anti-symmetric modes are significantly smaller than their symmetric counterpart; thus, for low S/D, it is expected that the resonance loop may be closed for even modes, and not for odd modes. Interestingly, the acoustic spectrum displays a “peak-doubling” feature as the spacing is increased. As the wavenumbers of even and odd Kelvin-Helmholtz modes predicted from linear stability analysis are almost indistinguishable for these conditions (see (Morris, 1990), for instance), it is impossible to argue for the dominance of either symmetry based on information about this instability wave alone. Thus, it is possible that the increase in the bands of existence of odd guided jet modes will lead to the support of new resonance cycles in the flow. Even though Figure 2 suggests that the bands of existence of guided jet modes of different symmetries may affect the coupling mechanism of the jets, single microphone measurements alone are not sufficient to extract the symmetry related to each screech tone. This is evaluated by means of a spectral proper orthogonal decomposition applied to the schlieren data, which is performed in the next section.

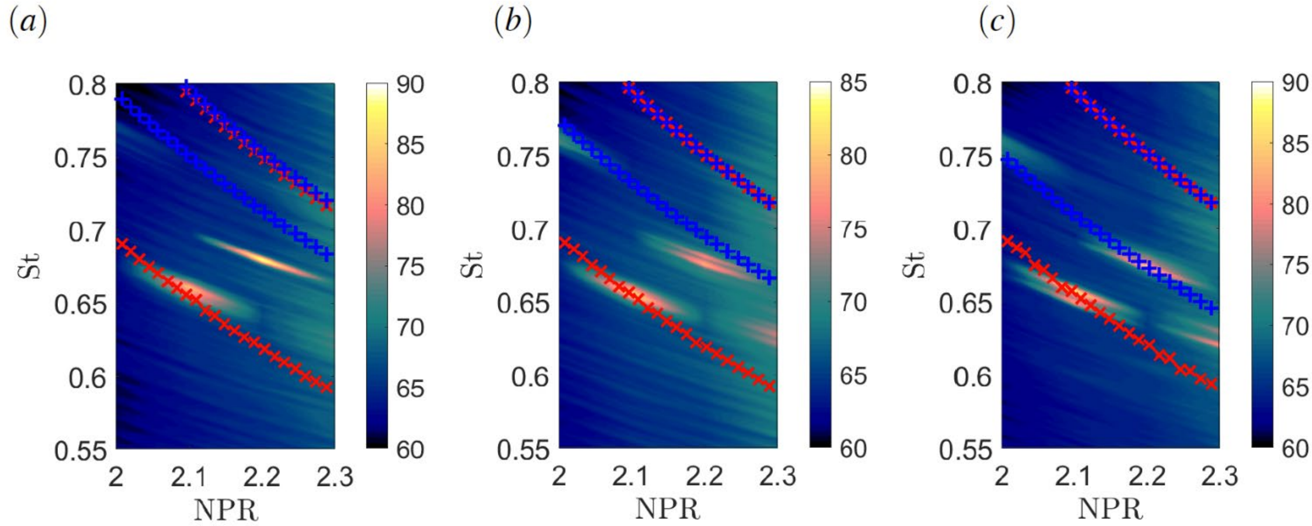

Figure 2: Sound pressure level (dB/St) from a twin jet system for $S/D = 2$ (a), 3 (b) and 6 (c). For all cases, both the cut-on and cut-off frequencies of the guided jet modes predicted using the TJVS (which define their bands of existence) are shown for both symmetric ($\times$) and anti-symmetric ($+$) cases.

4.2 Coupled wavepackets from sym-SPOD

Results from the previous section suggest that the symmetry chosen by the resonance cycle can be strongly affected by the jet spacing. We will now focus on the identification of the coupling symmetries in each condition using sym-SPOD. Figure 3 shows sample SPOD spectra for both symmetries and $NPR = 2$, $S/D = 2$. A clear peak can be observed around the screech frequency for both cases (which is a consistent feature in all cases analysed), with a large separation between optimal and suboptimal modes. This suggests that, despite the low number of modes, SPOD may still capture the main features of the most energetic coherent structures in this jet. For this sample case, the energy of symmetric (even) modes is higher than its anti-symmetric (odd) modes. This points to a dominance of even modes for this condition. The presence of peaks for both even and odd modes may be related to a even-odd mode switching mechanism, similar to what is observed in the A1-A2 stages in axisymmetric single jets (Mancinelli et al. 2019), or to resonant conditions in which the jets are uncoupled. However, the schlieren images in the current study show that resonant modes in both jets are usually coupled. In most cases, a high-frequency peak is also observed in the spectrum, associated with the first harmonic of the screech tone. A low-frequency peak also arises, especially for the low S/D cases; these peaks are associated with a coupling between the jets (Rodriguez et al., 2018), but are not directly related to the screech phenomenon.

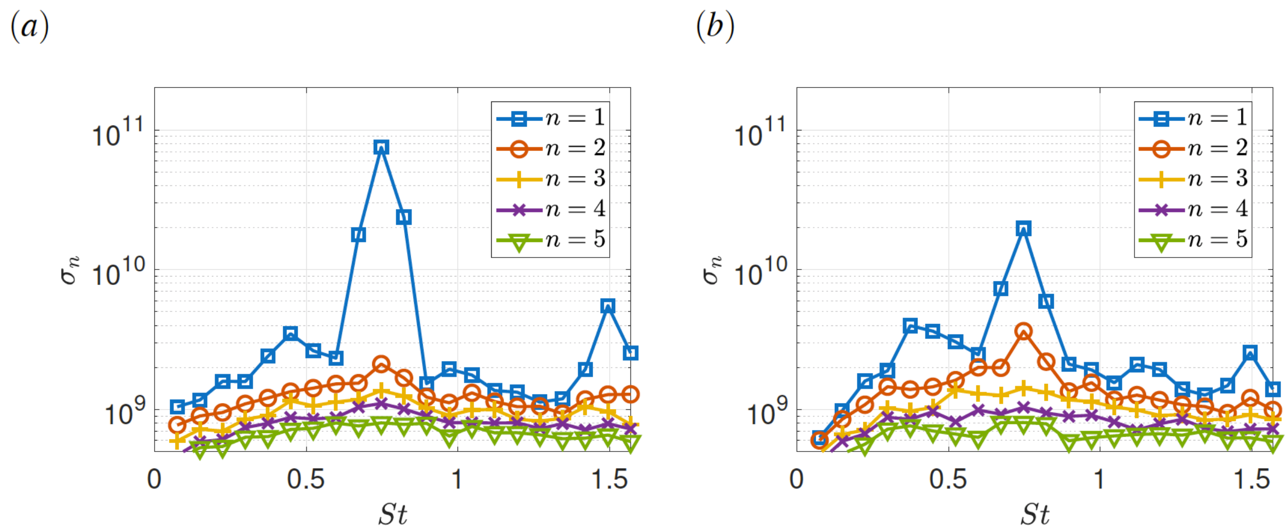

Figure 3: Energies of sym-SPOD modes for $NPR = 2$, $S/D = 2$ for even/symmetric (a) and odd/anti-symmetric (b) fields.

The effect of increasing the distance between the jets in the modes is seen in Figure 4, where the mode shapes are shown using saturated contours, to highlight the presence of the weaker waves (such as acoustic waves) in the flow. Figure 4 confirms that the flow is dominated by axisymmetric disturbances in each jet, as no phase jump is observed across the shear layer (at least in the early stages of the flow). In all cases, the presence of upstream-travelling waves is observed close to the nozzle (represented by the wavefronts pointing towards the nozzle within the first dimaters of the jet), highlighting the resonating nature of these jets; in most of them, strong acoustic waves (possibly associated with Mach wave radiation) are also observed in the region $y > S/2$. As expected, all anti-symmetric modes have zero amplitudes at $y = 0$, which leads to some distinct features on the dominant structures, when compared to the symmetric ones. Such differences are clearer as the jets spacing increases, having a clear effect on the amplitude of the waves in the interjets region ($y < S/2$). Figure 4 shows that the coupling still exists even for $S/D = 6$, and that causes the appearance of an acoustic wave travelling downstream, parallel to the jets. This wave may have been generated by interference between the radiation of both jets, or by reflection of upstream-travelling waves generated by both jets at the facility wall. This represents a significant change in the pressure distribution of these jets, compared to the single jet case, and can be linked to the fact that twin jets may still be weakly coupled even at large S/D (Raman and Taghavi, 1998).

Figure 5 shows the relative energy of the optimal screech modes from sym-SPOD, defined as the ratio between the maximum SPOD energy from even and odd modes ($\sigma_{max,rel} = \sigma_{max,even}/\sigma_{max,odd}$) in a logarithmic scale; datapoints in the y-negative region of the plot are dominated by odd modes, and the ones in the y-positive region are dominated by even modes. For $S/D = 2$ and most values of NPR, the flow is dominated by even modes (except for $NPR = 2.1$, where a competition between odd and even modes seems to occur). This is in-line with the TJVS model, which shows that the low range of existence of upstream modes for such low spacing may hinder the closure of the resonance loop. For large spacings ($S/D = 6$), no clear dominance is observed, and the jets have similar probability of coupling either symmetrically or anti-symmetrically, or even of being uncoupled. For $S/D = 3$, the jets couple strongly at an odd mode for $NPR = 2.1$. This may be connected to the higher spatial amplification rate of anti-symmetric modes at moderate spacings, shown in (Rodriguez et al., 2018). As the resonance loop may be closed in both symmetries, the growth rate of the KH mode close to the position where resonance is closed may affect which mode will be dominant.

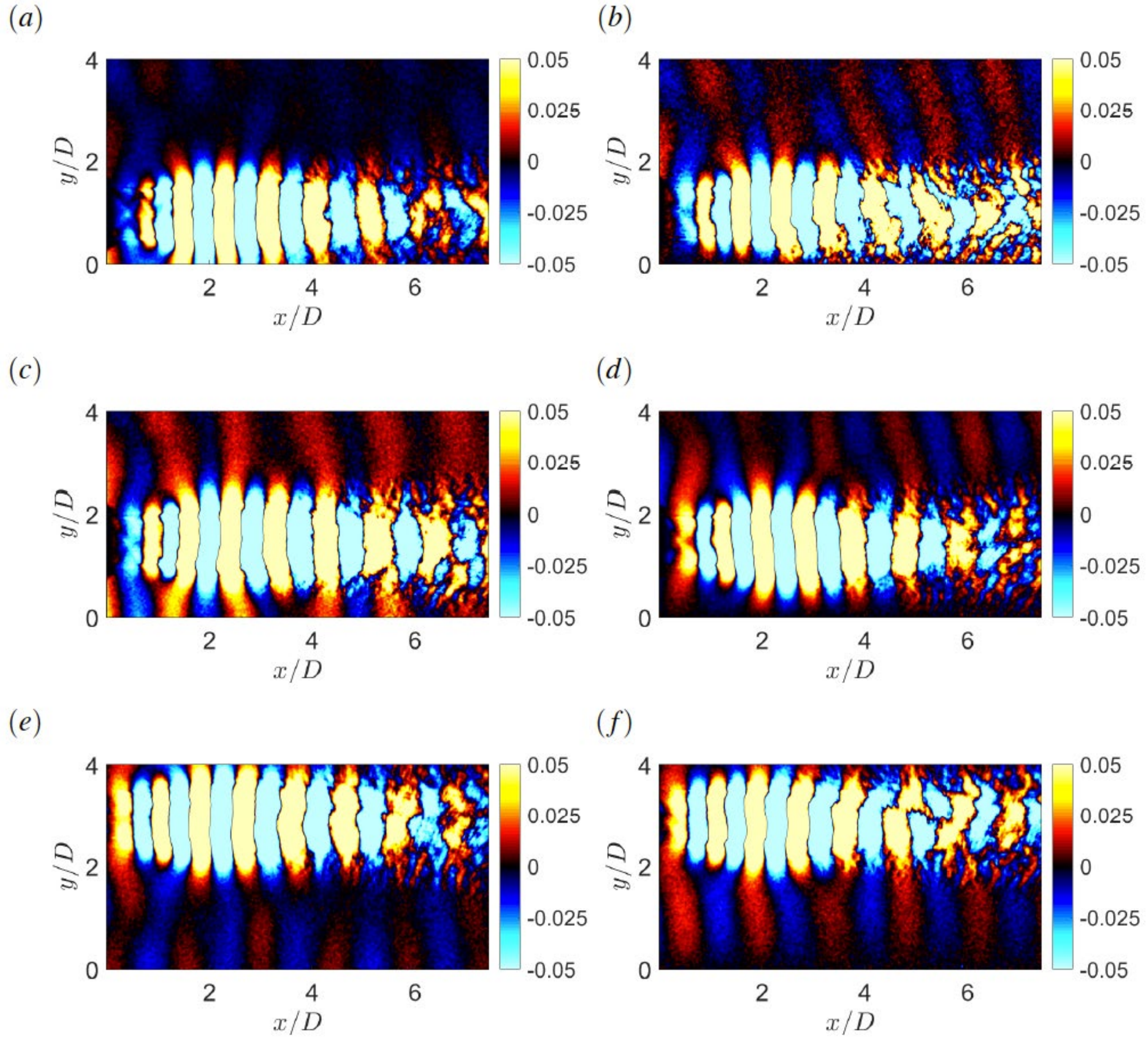

Figure 4: Shapes of dominant sym-SPOD modes at the screech frequency for $NPR = 2$, $S/D = 2$ (a,b), $S/D = 3$ (c,d) and $S/D = 6$ (e,f). Both even (a,c,e) and odd (b,d,f) modes are shown. All modes are normalised by their maximum, and fields are presented using saturated contours.

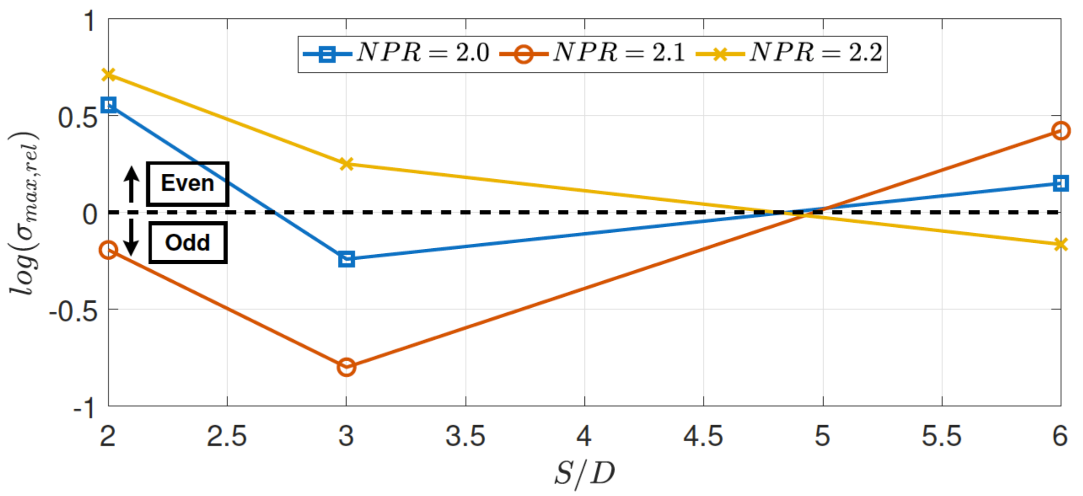

Figure 5: Relative energy of dominant modes at the screech frequency shown in log-scale ($\sigma_{max,rel} = \sigma_{max,even}/\sigma_{max,odd}$) for each operating condition.

5 CONCLUSIONS

A study of the coupling mechanism of wavepackets in twin-jet systems was performed. Analysis of the frequency bands of existence of guided jet modes using a twin-jet vortex sheet model shows that most screech tones are found in frequencies where these waves are supported by the flow, suggesting that these waves might be responsible for closing the resonance loop in multi-jet configurations. This model also shows that anti-symmetric modes are mitigated if the jets are brought closer to each other; thus, an increasing in spacing may lead to the appearance of new tones in the spectrum, which is also observed in the experimental data. The dominant symmetry for each condition was obtained by means of a symmetry-imposed spectral proper orthogonal decomposition applied to schlieren data. This method leads to the most energetic coherent structures in specific frequencies and their relative energy. The results show that mode shapes are modified with spacing, especially in the interjets region. While the number of schlieren images is not sufficient to provide the exact tone frequency, the relative energy of the dominant mode close to the screech frequency allows us to identify which symmetry is dominant in each condition. It is shown that symmetric modes are usually more energetic for very low spacings, and no clear dominance is found for large spacings. The reason for appearance (and dominance) of odd modes for $NPR = 2.1$ and low spacings is still unclear. This may be due to effects related to the merging of the jets and the development of the shear layer, which are not considered in the TJVS model. Application of a finite thickness model for linear stability analysis and other methods to consider the streamwise variation of the mean flow might lead to results more comparable with experiments. Also, with the aid of prediction models, we might also obtain a clearer picture of the symmetry-locking mechanism in screeching twin-jet systems.

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