Sound radiation from a plate immersed in water near the free surface

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ABSTRACT
The results on the acoustic response of a heavy fluid-loaded baffled plate excited by a harmonic point force are presented. The displacements of the fluid-loaded plate are decomposed on the basis of the in-vacuo plate modes and the radiated acousic waves are determined solving the Helmholtz equation in a fluid. The Green's function for the acoustic waveguide domain formed by the baffled plate and the free surface is modelled by the source-image method. Predictions for the radiated sound power from the plate and pressure spectra are calculated for varying depths of the free surface and compared against results from an unbounded domain to infer the effect of the free surface on the acoustic response of the plate. The proposed analytical model is verified by comparison with finite element simulations.

1 INTRODUCTION
Modelling the vibroacoustic behavior of elastic structures submergerd in heavy fluid, especially in water, is an important fluid-structure coupling problem that is continuously a major focus in acoustics research. Analytical models present an efficient and flexible framework to parameterize aspects of an investigation. In relation to the analysis of plates, the simple structure is found to be relevant as it presents a first-order approximation to more complex problems of submerged structures such as a cylindrical shell. Studies on the fluid-plate interaction are mainly concerned about the excited response from deterministic forces (Sandman 1977, Sun et al. 2015) and stochastic excitations, such as with interaction of a turbulent boundary layer (Karimi et al. 2020a, Karimi et al. 2020b). However, there is a lack of literature that introduces a free surface to the fluid domain. This is relevant to structures for shallow water applications. The present analytical approach aims to investigate how a free surface affects the vibroacoustic behavior of a forced baffled plate.

A modal matrix approach, similar to which has been previously applied in literature (Sandman 1977, Gu and Fuller 1993), is applied to the fluid-plate system to calculate the modal coefficients of the plate transversal motion W_{mn}. However, with the inclusion of a free surface as a pressure release boundary condition, the resulting fluid loading impedance expression requires an alternate Green's function to encompass the effects of the wave guide domain.

For the acoustic response of the plate, the field pressure can be evaluated with the following expressions

$$p(\mathbf{R}) = \rho_0 \omega^2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \iint \varphi_{mn} G_{WG}(R|R_1)\, dx_1 dy_1 \tag{1}$$

$$G(\mathbf{R}|\mathbf{R}_0) = \frac{1}{2\pi}\left(\frac{e^{-jk_f|\mathbf{R}-\mathbf{R}_0|}}{|\mathbf{R}-\mathbf{R}_0|} - 2\sum_{l=0}^{\infty}(-1)^l \frac{e^{-jk_f|\mathbf{R}_l'-\mathbf{R}_0|}}{|\mathbf{R}_l'-\mathbf{R}_0|}\right), \text{ where } |\mathbf{R}_l'|^2 = |\mathbf{R}_0|^2 + (2H + 2lH)^2, \tag{2}$$

G is the Green's function solution to the Helmholtz equation and can be modelled by the source-image method (Wu 1994). It is also recognized that the Green's function presents a standing waves criteria, where the wave-lengths of outgoing acoustic waves $\lambda = 4H/(2N+1)$ for $N = 0,1,2,3..$, in which at certain frequencies for a given height of a free surface, the vibroacoustic behavior of the plate may be slightly altered.

2 ACOUSTIC RESULTS
The technique is implemented in MATLAB. The plate and fluid specifications are summarized in Table 1 and the magnitude of the force is F_0 = 10 N applied at the centre of the plate (a/2,b/2,0). In Figure 1, it is observed that the free surface effect manifests as the standing waves criteria and slightly alters the acoustic response of the plate.

Table 1: Dimensions and material properties of the plate and fluid properties

Parameter	Value
Young's Modulus E (GPa)	68
Poisson's ratio, ν	0.3
Density of plate, ρ_0 (kg/m^3)	2740
Density of fluid, ρ_p (kg/m^3)	1026
Speed of sound, c_0 (m/s)	1500
Plate length, a (mm)	600
Plate width, b (mm)	525
Plate thickness, h (mm)	2.4
Structural damping, η	0.01
Fluid damping, η_f	0.0001

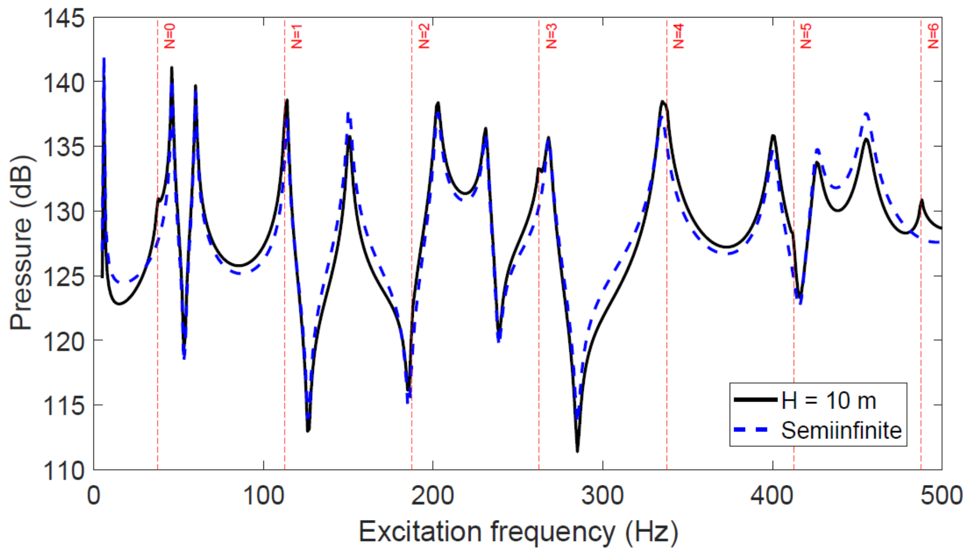

Figure 1: Acoustic pressure at an observation point of (a/2,b/2,5) for a plate in a heavy fluid domain with a far free surface (H = 10 m), and in a semiinfinite fluid domain

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